Misdirected Meteor

A meteor of mass $m$ is in a circular orbit about an airless planet of radius $R$ and mass $M$ at an altitude of $h = 3R$ above the planet’s surface. The meteor suddenly undergoes a head-on collision with a small piece of space debris. As a result of the collision, the meteor loses half of its kinetic energy without changing its direction of motion or its total mass.

Answer the following questions about the meteor and its orbit after the collision.

You may find it useful to recall that the total energy of an elliptical orbit is $E = -GMm/2a$ where $a$ is the semi-major axis. You may also wish to recall Kepler’s third law: $T^2 = (4\pi^2/GM)a^3$.

(2 points) (a) Find the kinetic energy $K$, the potential energy $U$, and the angular momentum $L$ of the meteor immediately after the collision.

(2 points) (b) What is the shape of the meteor’s orbit? Use the results from part (a) to justify your answer. Make a sketch of the orbit and indicate the meteor’s initial position. (i.e., the position of the meteor at the time of the collision).

(2 points) (c) Find the minimum distance $h_{\text{min}}$ of the meteor from the surface of the planet, and its maximum speed $v_{\text{max}}$. Hint: Use conservation of energy and angular momentum.

(2 points) (d) Find the time it takes for the meteor to travel from its position in part (c) back to the position where it had the collision.