Ph 2a

COMPLEX NUMBERS

A complex variable can always be written in terms of 2 real variables $x$ and $y$, and $i = \sqrt{-1}$ as:

$$z = x + iy$$

Its complex conjugate is

$$z^* = x - iy$$

If you invert these you get the real and imaginary parts of $z$:

$$\text{Re}\{z\} = x = \frac{1}{2}(z + z^*)$$

$$\text{Im}\{z\} = y = \frac{1}{2i}(z - z^*)$$

The modulus squared of $z$ is:

$$zz^* = |z|^2 = (x + iy)(x - iy) = x^2 + y^2$$

Euler's Law, a very useful relation is:

$$e^{i\phi} = \cos \phi + i \sin \phi$$

from which we can deduce that:

$$|e^{i\phi}|^2 = e^{i\phi} (e^{i\phi})^* = e^{i\phi} e^{-i\phi} = e^0 = 1$$

So any complex number can be written also in terms of its modulus $\rho$ and phase $\phi$:

$$z = \rho (\cos \phi + i \sin \phi) = \rho e^{i\phi}$$

where

$$\rho = \sqrt{x^2 + y^2} = |z|$$

$$\phi = \tan^{-1}(y/x)$$

Graphical visualization of Complex Plane:

A useful trick for dealing with complex divisors:

$$z = a + ib$$
$$w = c + id$$

$$y = \frac{z}{w} = \frac{a + ib}{c + id} = \frac{w^* z}{w^* w} = \frac{c - id}{c^2 + d^2} a + ib$$

$$\frac{w^* z}{w^* w} = \frac{c - id}{c^2 + d^2}$$

$$y = \frac{ca + db}{c^2 + d^2} + i \frac{bc - da}{c^2 + d^2}$$

$$\rho$$

$$\phi$$

$z$

$x$

$y$