**Problem 4.16** A **hydrogenic atom** consists of a single electron orbiting a nucleus with $Z$ protons ($Z = 1$ would be hydrogen itself, $Z = 2$ is ionized helium, $Z = 3$ is doubly ionized lithium, and so on). Determine the Bohr energies $E_n(Z)$, the binding energy $E_1(Z)$, the Bohr radius $a(Z)$, and the Rydberg constant $R(Z)$ for a hydrogenic atom. (Express your answers as appropriate multiples of the hydrogen values.) Where in the electromagnetic spectrum would the Lyman series fall, for $Z = 2$ and $Z = 3$? **Hint:** There’s nothing much to **calculate** here—in the potential (Equation 4.52) $e^2 ightarrow Z e^2$, so all you have to do is make the same substitution in all the final results.

**Problem 4.17** Consider the earth-sun system as a gravitational analog to the hydrogen atom.

(a) What is the potential energy function (replacing Equation 4.52)? (Let $m$ be the mass of the earth, and $M$ the mass of the sun.)

(b) What is the “Bohr radius,” $a_g$, for this system? Work out the actual number.

(c) Write down the gravitational “Bohr formula,” and, by equating $E_n$ to the classical energy of a planet in a circular orbit of radius $r_o$, show that $n = \sqrt{r_o/a_g}$. From this, estimate the quantum number $n$ of the earth.

(d) Suppose the earth made a transition to the next lower level $(n - 1)$. How much energy (in Joules) would be released? What would the wavelength of the emitted photon (or, more likely, graviton) be? (Express your answer in light years—is the remarkable answer a coincidence?)

**Problem 4.19**

(a) Starting with the canonical commutation relations for position and momentum (Equation 4.10), work out the following commutators:

$$
[L_z, x] = i \hbar y, \quad [L_z, y] = -i \hbar x, \quad [L_z, z] = 0, \quad [L_z, p_x] = i \hbar p_y, \quad [L_z, p_y] = -i \hbar p_x, \quad [L_z, p_z] = 0.
$$

(b) Use these results to obtain $[L_z, L_z] = i \hbar L_y$ directly from Equation 4.96.

(c) Evaluate the commutators $[L_z, r^2]$ and $[L_z, p^2]$ (where, of course, $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$).

(d) Show that the Hamiltonian $H = (p^2/2m) + V$ commutes with all three components of $L$, provided that $V$ depends only on $r$. (Thus $H, L^2$, and $L_z$ are mutually compatible observables.)
Problem 4.25 If the electron were a classical solid sphere, with radius

\[ r_c = \frac{e^2}{4\pi \epsilon_0 mc^2} \quad [4.138] \]

(The so-called **classical electron radius**, obtained by assuming the electron's mass is attributable to energy stored in its electric field, via the Einstein formula \( E = mc^2 \)), and its angular momentum is \((1/2)\hbar\), then how fast (in m/s) would a point on the "equator" be moving? Does this model make sense? (Actually, the radius of the electron is known experimentally to be much less than \( r_c \), but this only makes matters worse.)

*Problem 4.27* An electron is in the spin state

\[ \chi = A \left( \frac{3i}{4} \right). \]

(a) Determine the normalization constant \( A \).

(b) Find the expectation values of \( S_x, S_y, \) and \( S_z \).

(c) Find the "uncertainties" \( \sigma_{S_x}, \sigma_{S_y}, \) and \( \sigma_{S_z} \). (Note: These sigmas are standard deviations, not Pauli matrices!)

(d) Confirm that your results are consistent with all three uncertainty principles (Equation 4.100 and its cyclic permutations—only with \( S \) in place of \( L \), of course).

*Problem 4.29*

(a) Find the eigenvalues and eigenspinors of \( S_y \).

(b) If you measured \( S_y \) on a particle in the general state \( \chi \) (Equation 4.139), what values might you get, and what is the probability of each? Check that the probabilities add up to 1. Note: \( a \) and \( b \) need not be real!

(c) If you measured \( S_y^2 \), what values might you get, and with what probabilities?

Problem 4.32 In Example 4.3:

(a) If you measured the component of spin angular momentum along the \( x \) direction, at time \( t \), what is the probability that you would get \( +\hbar/2 \)?

(b) Same question, but for the \( y \)-component.

(c) Same, for the \( z \) component.