Problem 4.2 Use separation of variables in cartesian coordinates to solve the infinite cubical well (or “particle in a box”):

\[ V(x, y, z) = \begin{cases} 0, & \text{if } x, y, z \text{ are all between } 0 \text{ and } a; \\ \infty, & \text{otherwise}. \end{cases} \]

(a) Find the stationary states, and the corresponding energies.

(b) Call the distinct energies \( E_1, E_2, E_3, \ldots \), in order of increasing energy. Find \( E_1, E_2, E_3, E_4, E_5, \) and \( E_6 \). Determine their degeneracies (that is, the number of different states that share the same energy). Comment: In one dimension degenerate bound states do not occur (see Problem 2.45), but in three dimensions they are very common.

(c) What is the degeneracy of \( E_{14} \), and why is this case interesting?

Problem 4.3 Use Equations 4.27, 4.28, and 4.32, to construct \( Y_0^0 \) and \( Y_2^1 \). Check that they are normalized and orthogonal.

Problem 4.13

(a) Find \( \langle r \rangle \) and \( \langle r^2 \rangle \) for an electron in the ground state of hydrogen. Express your answers in terms of the Bohr radius.

(b) Find \( \langle x \rangle \) and \( \langle x^2 \rangle \) for an electron in the ground state of hydrogen. Hint: This requires no new integration—note that \( r^2 = x^2 + y^2 + z^2 \), and exploit the symmetry of the ground state.

(c) Find \( \langle x^2 \rangle \) in the state \( n = 2, l = 1, m = 1 \). Warning: This state is not symmetrical in \( x, y, z \). Use \( x = r \sin \theta \cos \phi \).
Problem 4.14 What is the most probable value of $r$, in the ground state of hydrogen? (The answer is not zero!)

**Hint:** First you must figure out the probability that the electron would be found between $r$ and $r + dr$.

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Problem 4.15 A hydrogen atom starts out in the following linear combination of the stationary states $n = 2, l = 1, m = 1$ and $n = 2, l = 1, m = -1$:

$$\Psi(r, 0) = \frac{1}{\sqrt{2}} (\psi_{211} + \psi_{21-1}).$$

(a) Construct $\Psi(r, t)$. Simplify it as much as you can.

(b) Find the expectation value of the potential energy, $\langle V \rangle$. (Does it depend on $r$?) Give both the formula and the actual number, in electron volts.

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*Problem 4.38 Consider the three-dimensional harmonic oscillator, for which the potential is

$$V(r) = \frac{1}{2} m \omega^2 r^2.$$  \[4.188\]

(a) Show that separation of variables in cartesian coordinates turns this into three one-dimensional oscillators, and exploit your knowledge of the latter to determine the allowed energies. **Answer:**

$$E_n = (n + 3/2) \hbar \omega.$$  \[4.189\]

(b) Determine the degeneracy $d(n)$ of $E_n$. 