1. Solution:
   a. First we consider triples. For any fixed number, there are $\binom{5}{3}$ possible triples, because we could have triples of differing colors. Thus we have $8 \cdot \binom{5}{3}$ triples. Now we have pairs remaining. Of the pairs, there are 7 possible values left to choose, and their colors must be the remaining colors left. Thus there are $7 \cdot 8 \cdot \binom{5}{3}$ possible Full Houses.
   
   b. This obviously holds since $k$ and $l$ are symmetric.
   
   The left side counts the number of ways of choosing $k$ objects from a set of $n$, and then choosing $\ell$ objects from the remaining objects. We could have done this instead by picking the $\ell$ objects first, and then choosing $k$ from the remaining, which is the right side.
   
   c. There are $n$ people, and each can shake hands with either 0, 1, 2, $\ldots$, $n - 1$ people. If the set of handshakes that take place has size less than $n$, then by the Pigeon-Hole principle, two people shook the same number of hands. Otherwise, the set of handshakes is the set $\{0, 1, 2, \ldots, n - 1\}$. But this is impossible because the person who shook $n - 1$ hands shook everyone’s hand, so there can’t be someone who shook 0 hands.

2. Solution:
   Ask instructor (Solution to come).

3. Solution:
   a. Expand the LHS:

   $$LHS = \sum_{k=m}^{n} \frac{n!}{(n-k)!k!(k-m)!m!} = \sum_{k=m}^{n} \frac{(n-m)!}{(n-k)!(k-m)!m!} \cdot \frac{n!}{n!} = 2^{n-m} \binom{n}{m}$$

   b. The RHS counts the number of ways to choose two disjoint sets from $n$ objects, s.t. one set has cardinality $m$. It’s then equivalent to choose $k$ ($k > m$) objects from these $n$ objects first, and then partitioned these $k$ objects into two piles, s.t. one pile has $m$ objects, which is just LHS.

4. Solution:
   The set of surjections $S = (\bigcup_{i=1}^{n} A_{i})^{c}$, where $A_{i}$ denotes the set of mappings whose images do not contain $i$.

   By Inclusion-Exclusion Principle, we have:

   $$|S| = \sum_{j=0}^{n} (-1)^{j}|N_{j}|$$

   where $N_{j} := \sum_{|I|=j} N(I)$ and $N(I) := |\bigcap_{i \in I} A_{i}|$.

   Clearly, for $|I| = j$, $|N(I)| = (n-j)^{m}$, so $|S| = \sum_{j=0}^{n} (-1)^{j} \binom{n}{j}(n-j)^{m}$. □
5. Solution:

a. Since the chalkboard’s width is only 2, consider the first column of this chalkboard.

Case 1: there are just two $1 \times 1$ square tiles in the first column.
Remove these 2 tiles, we get a chalkboard of $2 \times (n - 1)$.
Case 2: there are one $1 \times 1$ square tile and one L-shaped tile in the first two columns.
Clearly, it has 4 ways to put the $1 \times 1$ square tile and then the corresponding L-shaped tile is also fixed.
Remove these 2 tiles, we get a chalkboard of $2 \times (n - 2)$.
Case 3: there are 2 L-shaped tiles in the first three columns.
Clearly, it has 2 ways to put these L-shaped tiles in the first three columns.
Remove these 2 tiles, we get a chalkboard of $2 \times (n - 3)$.
Combining these 3 cases, we get a recursion formula for $n \geq 3$:
$$T_n = T_{n-1} + 4T_{n-2} + 2T_{n-3}$$

b. Consider the generating function $f(x) = \sum_{n \geq 0} T_n x^n$, with initial conditions $T_0 = 1, T_1 = 1, T_2 = 5$, we have:
$$f(x) = \sum_{n \geq 0} T_n x^n = 1 + x + 5x^2 + \sum_{n \geq 3} (T_{n-1} + 4T_{n-2} + 2T_{n-3})x^n$$
$$= 1 + x + 5x^2 + x(f(x) - x - 1) + 4x^2(f(x) - 1) + 2x^3f(x)$$
$$f(x) = \frac{1}{1 - x - 4x^2 - 2x^3} = \frac{1}{1 + x - \frac{2x}{2x^2 + 2x - 1}} = \frac{1}{1 + x - \frac{3 + \sqrt{3}}{2}x - \frac{3 - \sqrt{3}}{2}}$$
Expand RHS and we get
$$T_n = (-1)^n + \frac{\sqrt{3}}{3}[(1 + \sqrt{3})^n - (1 - \sqrt{3})^n]$$
It’s then easy to check that $T_3 = 11, T_4 = 33, T_5 = 87$. □