Let $R$ be a ring, with $1 \in R$.

1. When is $R$ a simple module over itself?

2. (a) List all non-isomorphic semisimple rings with 81 elements.
   (b) Which of these are commutative?

3. Let $R$ be a left Artinian ring, and let $I$ be an ideal of $R$. Assume that $I^2 = (0)$ implies that $I = (0)$ for all $I$. Prove that $R$ is semisimple.


5. Let $G$ be a group and let $H$ be a subgroup of $G$ of finite index. Let $K$ be a field such that the characteristic does not divide $[G : H]$.
   Let $V$ be a $KG$-module with a submodule $W$ and let $U_o$ be a $KH$-module such that $V = W \oplus U_o$.
   Show that there exists a $KG$-submodule $U$ of $V$ such that $V = W \oplus U$.

6. Let $K$ be a field and define
   \[ S := \left\{ \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} : a, b, c \in K \right\}. \]
   How many simple modules does $S$ have? Is $S$ a semisimple ring?