1. Compute the Galois groups of
(a) \(x^3 - 15\) over \(\mathbb{Q}\),
(b) \(x^3 - 21x + 7\) over \(\mathbb{Q}\),
(c) \(x^3 - 15\) over \(\mathbb{Q}(\sqrt{3})\),
(d) \(x^4 - 13\) over \(\mathbb{Q}, \mathbb{Q}(\sqrt{13}), \mathbb{Q}(\sqrt{-13}), \mathbb{Q}(i)\).

2. Let \(K = \mathbb{C}(t)\), where \(t\) is an indeterminate. Calculate the Galois groups over \(K\) of the splitting fields of
(a) \(X^3 + X + t\),
(b) \(X^3 + tX + 1\),
(c) \(X^3 + t^2X - t^3\).

3. Let \(f(x) = x^4 + ax^2 + b \in \mathbb{Q}[x]\). Let \(\pm \alpha\) and \(\pm \beta\) be the roots of \(f\), let \(K\) be the splitting field over \(\mathbb{Q}\) and let \(G\) be the Galois group. Show that
(a) \(G\) is isomorphic to the dihedral group of order 8.
(b) \(G \cong \mathbb{Z}/4\mathbb{Z}\) if and only if
\[
\frac{\alpha}{\beta} - \frac{\beta}{\alpha} \in \mathbb{Q}.
\]
(c) \(G \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}\) if and only if either \(\alpha \beta \in \mathbb{Q}\) or \(\alpha^2 - \beta^2 \in \mathbb{Q}\).