1. Let $p$ be a rational prime, and let $F$ be the property that

For all $a \in \mathbb{Z}$, \( \left( \frac{a}{p} \right) = -1 \iff a \text{ is a primitive root modulo } p. \)

Show that $F$ holds if and only if $p$ is a Fermat prime.

2. Let $n$ be a composite integer. Show that \((n-1)! \equiv -1 \mod n\).

3. Let $p$ be a rational prime. Show that \((p-1)! \equiv -1 \mod p\). Conclude that checking the value of \((m-1)! \mod m\) is a test for the primality of $m$. Is this a reasonable test? Check that 11 is prime using this test.

4. Why is ideal theory for fields an uninteresting topic?

5. Let $n$ be a rational integer greater than 2. Show that there are no primitive roots modulo $2^n$. 