1. Show that all imaginary quadratic fields have only finitely many units, and determine these explicitly.

2. Prove that any positive definite binary integral quadratic form of discriminant 5 is equivalent over \( \mathbb{Z} \) to either \( x^2 + 5y^2 \) or \( 2x^2 + 2xy + 3y^2 \). Show that these two forms are not equivalent to one another over \( \mathbb{Z} \).

3. Show that there are infinitely many units in \( \mathbb{Q}(\sqrt{2}) \), and hence exhibit three solutions to the equation

\[
x^2 - 2y^2 = 1, \ x, y \in \mathbb{Z}.
\]

Hint: find a unit in \( \mathbb{Q}(\sqrt{2}) \) which is not \( \pm 1 \), and show that it is not a root of unity.

4. Let \( A \) be the adeles of \( \mathbb{Q} \) and let \( A^f \) be the subring of the adeles whose real coordinate is 0. We see that \( \mathbb{Q} \) embeds diagonally into \( A^f \). Prove that the image of \( \mathbb{Q} \) is dense in \( A^f \).

5. Let \( S \) be a finite set of places of \( \mathbb{Q} \), and let \( A_S \) be the subring of \( A \) consisting of the elements of \( A \) whose \( v \)-coordinate is 0 for all \( v \in S \). Show that, if \( S \) is not empty, then \( \mathbb{Q} \) is dense in \( A_S \).

6. Let \( m \) be a squarefree integer. Show that the fields \( \mathbb{Q}(\sqrt{m}) \) are pairwise distinct, by considering the equation \( \sqrt{m} = a + b\sqrt{n} \) or otherwise.