Cosmology (Ay127), Spring 2010

Problem Set 1

Due: In class, first class of week 2

1. Show that the two forms of the Friedmann equation lead to identical
dynamics. Do not assume that the matter is only pressureless.

2. Although general relativity is not required for this class, here’s a simple ex-
cercise with metric that doesn’t really require relativity. Find a coordinate
transformation that shows that the Milne spacetime (i.e., the FRW space-
time with Friedmann equation $H^2 \propto a^{-2}$) is equivalent to a Minkowski
spacetime. Explain what is going on.

3. The age $t_0$ of the Universe in the standard cosmological model depends
on the current value of the Hubble parameter, $H_0 = 100\, h$ km/sec/Mpc,
as well as on $\Omega_0$, the current density in units of the critical density. In
class, we showed that if $\Omega_m = 1$ and $\Omega_{\Lambda} = 0$, then $t_0(h) = 6.7\, h^{-1}$ Gyr.
(a) Your assignment is to generalize this result and derive an expression
for the age of the Universe for $\Omega_m > 1$ and for $\Omega_m < 1$, both for $\Omega_{\Lambda} = 0$.
(This shouldn’t be too tricky—the answers are in the books. But still, you
should derive the equations yourself.) Then, plot contours for $t_0 = 10$ Gyr,
13 Gyr, and 17 Gyr on the $\Omega_m - h$. You may do this either by sketching
the contours by hand, or you may generate such a plot with Mathematica,
C, Fortran, or anything else, if you’re so inclined. (b) Then, make the
analogous plots, but for $\Omega_m + \Omega_{\Lambda} = 1$ (and restricting to $\Omega_m < 1$). (c)
Stellar astrophysicists believe that the oldest stars are around 10–20 Gyr.
If the correct value is somewhere around 14 Gyr, your plots should show
you for which values of $\Omega_m$ and $h$ there might be consistency. (d) A few
years ago, some astronomers found a galaxy at a redshift $z = 1.5$ with a
spectrum well fit by stellar-population model with an age 3.5 Gyr. Draw
on your plots the regions of the $\Omega_m$-$h$ parameter space ruled out by this
measurement.