PHYSICS 1c Prac – QUIZ 2

This is an OPEN BOOK quiz with the following limitations: You may consult only your textbook (Serway & Jewett), notes that you have taken in recitation or lecture, course handouts, and homeworks. No other references are allowed. Simple algebraic calculators are allowed. Unless otherwise stated, you should provide two significant figures if/when a numerical answer is requested.

The time limit is 2.5 HOURS, in one continuous sitting.

The quiz is DUE Monday, May 4 at noon and should be turned into the lock boxes outside of 201 E. Bridge. Late quizzes will not be accepted for credit except by prior arrangement with your TA in extraordinary circumstances.

There are three problems on pages 3 - 5 for a total of 18 points.

DON'T FORGET TO WRITE YOUR NAME and SECTION NUMBER clearly on the front of your exam.

As always, you are bound by the Honor Code in all matters concerning this exam.

Do not proceed to the next page until you are ready for the 2.5 hour clock to begin.
GOOD LUCK!
**Problem 1.** 6 points total

A circular wire ring of radius $a$ is in an external uniform magnetic field $B$. The electrical resistance of the ring is precisely zero and the thickness of the wire is negligible. The direction $n$ perpendicular to the ring is inclined relative to the magnetic field by an angle $\theta$ (which remains fixed). Prior to the present moment ($t = 0$) the magnetic field has been constant and no current is flowing in the ring. (You need not worry how this state of affairs was established.) The situation appears thusly:

At $t = 0$ the magnetic field begins to increase (in magnitude only). After a time $\Delta t$ the magnetic field has reached the value $B + \Delta B$ whereupon it stops changing and is once again constant.

**a)** 2 points - What is the current flowing in the ring once the field has reached its new value? You may assume that the self-inductance of the ring is $L$.

**b)** 2 points - What is the magnetic moment $\mathbf{m}$ of the ring after the field has stopped changing? (You need to find both its magnitude and direction.)

**c)** 2 points - What is the torque on the ring after the magnetic field has stopped changing? Again, be sure to specify both its magnitude and direction.
Problem 2. 6 points total

Consider the circuit shown below. Before the switch is closed at $t = 0$, capacitor $C_1$ is charged to a voltage $V_0$ while capacitor $C_2$ has no charge on it. The object of this problem is to determine what happens in the circuit after the switch is closed and current begins to flow. (Note: there is no mutual inductance between the two coils.)

![Circuit Diagram]

a) 2 points - Use Kirchoff’s Law to find an equation relating the current $I$ to the voltages $V_1$ and $V_2$ across the two capacitors, $C_1$ and $C_2$ and the inductances $L_1$ and $L_2$.

b) 2 points - Find the differential equation which governs the time dependence of the current $I(t)$.

Keep in mind that if charge $Q$ resides on the "top" plate of a capacitor (and of course $-Q$ resides on the bottom plate) then $dQ/dt$ gives the current which flows on to the top plate.

c) 2 points - Find the frequency of oscillations of the current. Express the answer in terms of the inductances $L_1$ and $L_2$ and the capacitances $C_1$ and $C_2$. 
**Problem 3.** 6 points, total.

Two identical $LC$ circuits are coupled by a mutual inductance $M$ as shown below.

![Diagram of coupled LC circuits with mutual inductance M]

With this set up, the voltages across the inductors are related to the currents in this way:

\[
V_1 = L \frac{dI_1}{dt} + M \frac{dI_2}{dt} \\
V_2 = L \frac{dI_2}{dt} + M \frac{dI_1}{dt}
\]

a) 2 points - Let $I_1$ and $I_2$ denote the currents flowing in the two loops, with 'positive' current defined by the arrows in the diagram. Find the differential equations which determine the time variation of these currents. Be careful about the signs of the various terms.

As in the previous problem, remember that if charge $Q$ resides on the "top" plate of a capacitor (and of course -$Q$ resides on the bottom plate) then $dQ/dt$ gives the current which flows on to the top plate.

b) 2 points - There are two particularly simple situations in this problem. In one the currents are equal: $I_1 = +I_2$. In the other the currents are equal in magnitude but opposite in sign: $I_1 = -I_2$. Show that each of these two cases leads to a simple oscillatory time dependence of the current. Find the associated natural resonant frequencies, $\omega_+$ and $\omega_-$, for these two cases. How do they compare to the frequency $\omega_0 = \sqrt{1/LC}$ of the individual loops when $M = 0$?

c) 2 points - In general, there are solutions to the circuit equations for which neither of the conditions described in **b)** hold true. Show that, nevertheless, the SUM of the currents, $I_1 + I_2$, always oscillates at frequency $\omega_+$, while the DIFFERENCE of the currents, $I_1 - I_2$, always oscillates at $\omega$. 