Math 116a Homework # 1
Due: 10-11-01

Problems labeled “CL” are from the textbook (Cori and Lascar), so “CL 1.16” means problem 16 from chapter 1 of the book. Please do not read the solutions for these problems in the textbook until you have completed the assignment.

1. CL 1.16

2. If $\varphi$ and $\psi$ are two formulas, we let $\varphi|\psi$ abbreviate $\neg(\varphi \land \psi)$. This is referred to as the Sheffer stroke (and in logic circuits is known as a nand gate).
   
   (a) Write the truth table for $|$.  
   (b) Show that $\neg P_1$ is logically equivalent to $P_1|P_1$ and that $P_1 \lor P_2$ is logically equivalent to $(P_1|P_1)|(P_2|P_2)$. This shows that $|$ is a complete binary connective: any other connective can be expressed in terms of it.  
   (c) Determine all complete binary connectives. There is precisely one other one; determine its truth table and show that there are no others.

3. Show that none of the following sets of connectives is complete: \{\neg, \leftrightarrow\}, \{\leftrightarrow, \land\}, \{\leftrightarrow, \lor\}.

4. Recall that if $S$ is a set of formulas, then $S \vdash^* \varphi$ ($S$ logically implies $\varphi$) means that any truth assignment making all of the formulas in $S$ true also makes $\varphi$ true.
   
   (a) Show that $S \cup \{\varphi\} \vdash^* \psi$ if and only if $S \vdash^* (\varphi \Rightarrow \psi)$.  
   (b) Show that if $S \vdash^* \varphi$ or $S \vdash^* \psi$ then $S \vdash^* (\varphi \lor \psi)$. Is the converse true?

5. CL 1.6

6. CL 1.12