Ma/CS 6c  
Assignment #1  
Due Tuesday, April 8 at 1 p.m.

(40%) 1*. (a) Prove the correctness of the following algorithm for recognizing when a given string $S$ is a $P$-wff:

If $S = s_1 s_2 \ldots s_n$, compute $w(s_n), w(s_n) + w(s_{n-1}), \ldots, w(s_n) + w(s_{n-1}) + \cdots + w(s_1) = w(S)$. If all these sums are $\geq 1$ and $w(S) = 1$, then $S$ is a $P$-wff; otherwise, it is not. (Recall that $w(p) = 1$, $w(\neg) = 0$, $w(\ast) = -1$, if $\ast = \land, \lor, \Rightarrow$.)

Apply this algorithm to the strings:

(i) $qp \Rightarrow \neg ttr \Rightarrow \land \neg stuv$;
(ii) $\Rightarrow \Leftarrow \land p q \lor \neg p \neg q s$.

If any of these strings is a $P$-wff, write down the corresponding wff.

(b) Formulate an analogous algorithm for recognizing $RP$-wff.

(40%) 2. Show that unique readability holds for the formal language described below (whose grammatically correct strings we call $R$-wff):

(i) Symbols: $p_0, p_1, \ldots, \neg, \land, \lor, \Rightarrow, \Leftarrow$;
(ii) Rules: (a) Each $p_i$ is a $R$-wff;

(b) If $A$ is a $R$-wff, so is $\neg A$.

(c) If $A, B$ are $R$-wff, so are $A \land B, A \lor B, A \Rightarrow B, A \Leftarrow B$).

(20%) 3. Define the following function

$$r : \text{wff} \to \{0, 1, 2, \ldots\}$$

by recursion:

$$r(p) = 0$$

$$r(\neg A) = r(A) + 1$$

$$r((A \ast B)) = \max\{r(A), r(B)\} + 1.$$ 

Compute $r(A)$ for a couple of examples of your choice. What does $r(A)$ mean in terms of the parse tree $T_A$ of the wff $A$?

(20%) 4. Consider the following kinds of gates:
Every wff using only \( \land, \lor, \neg \) corresponds to a circuit built out of these gates. For example, the wff \(((p \land q) \lor \neg r)\) corresponds to the circuit:

\[
\begin{array}{c}
p \\
q \\
r \\
\end{array} \quad \begin{array}{c}
p \land q \\
 \neg r \\
\end{array} \quad ((p \land q) \lor \neg r)
\]

and \(((q \lor p) \land (\neg p \land r))\) corresponds to the circuit:

\[
\begin{array}{c}
r \\
p \\
q \\
\end{array} \quad \begin{array}{c}
\neg p \\
\neg p \land r \\
(q \lor p) \\
((q \lor p) \land (\neg p \land r))
\end{array}
\]

(a) Construct a circuit corresponding to the wff:

\[((p \land q) \lor (p \land (\neg q \land r))).\]

(b) Find a wff corresponding to the circuit: