Review 3

1. Evaluate line integrals:

(a) \( \int_C x^2 y \, ds \), where \( C \) is parameterized by \( \vec{r}(t) = (\cos t) \vec{i} + (\sin t) \vec{j} + t \vec{k}, 0 \leq t \leq \frac{\pi}{2} \).

(b) \( \int_C x^3 \, ds \), where \( C \) is the portion of the unit circle in the first quadrant oriented counterclockwise.

(c) \( \int_C (x \, dx + y \, dy + z \, dz) \), where \( C \) is the line segment from \((0,1,0)\) to \((1,2,3)\).

(d) \( \int_C (y \, dx - x \, dy) \), where \( C \) is parameterized by \( \vec{r}(t) = (\ln t) \vec{i} + t \vec{j}, 1 \leq t \leq 2 \).

2. Find the work done by the force

\[
\vec{F} = \frac{-x \vec{i} - y \vec{j} - z \vec{k}}{(x^2 + y^2 + z^2)^{3/2}}
\]

when the object moves the point \((1,0,0)\) to the point \((1,1,1)\).

3. Using Green’s theorem, compute line integrals:

(a) \( \int_C (ye^x \, dx + \cos(x^2) \, dy) \), where \( C \) is the square with vertices \((\pm 1, \pm 1)\) oriented counterclockwise.

(b) \( \int_C (2y \, dx + x \, dy) \), where \( C \) is the leaf of \( r = \cos \theta \), \(-\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6} \), oriented clockwise.

4. Using line integrals, compute the area of the region bounded by \( \vec{r}(t) = (t^3 + t + 1) \vec{i} + (1 - t^2) \vec{j}, -1 \leq t \leq 1 \) above and by the \( x \)-axis below.

5. Compute surface integrals:

(a) \( \iint_\Sigma \frac{x^2}{x^2 + y^2} \, dS \), where \( \Sigma \) is the part of the paraboloid \( z = x^2 + y^2 \) below the plane \( z = x \).

(b) \( \iint_\Sigma (x + y) \, dS \), where \( \Sigma \) is the triangle with vertices \((0,0,1), (2,0,0), (0,1,0)\).

6. Evaluate \( \iint_\Sigma (\vec{F} \cdot \vec{n}) \, dS \) where

(a) \( \vec{F} = x \vec{i} + y \vec{j} + z \vec{k}; \Sigma \) is the part of the surface \( z = x^2 + y^2 \) inside of the cylinder \( x^2 + y^2 = 1 \); \( \vec{n} \) is directed upward.

(b) \( \vec{F} = \vec{i} + z \vec{j} + x \vec{k}; \Sigma \) is the portion of the cylinder \( x^2 + z^2 = 1 \) with \( 0 \leq y \leq 1 \); \( \vec{n} \) is directed outward.

7. Compute \( \int_C (x^2 \, dx + x^2 \, dy + y^2 \, dz) \), where \( C \) is the intersection of the sphere \( x^2 + y^2 + z^2 = 2 \) and the plane \( z = 1 \), and \( C \) has counterclockwise orientation as viewed from above.

8. Compute \( \int_C ((y+2z) \, dx + (3x+z) \, dy + (x+y) \, dz) \), where \( C \) is the intersection of the surfaces \( z = x^2 + 2y^2 \) and \( z = 1 - 2y^2 \), and \( C \) has clockwise orientation as viewed from above.

9. Compute \( \iint_\Sigma (\vec{F} \cdot \vec{n}) \, dS \) where \( \vec{F} = xy \vec{i} + z \vec{j}, \Sigma \) is the boundary of the portion of the ball \( x^2 + y^2 + z^2 \leq 1 \) in the first octant, and \( \vec{n} \) is directed outward.

10. Compute \( \iint_\Sigma (\vec{F} \cdot \vec{n}) \, dS \) where \( \vec{F} = x^2 \vec{i} + y \vec{j} + z \vec{k}, \Sigma \) is the boundary of the solid region which is bounded by \( z = x^2 + y^2 \) below and by \( z = 2 - x^2 - y^2 \) above, and \( \vec{n} \) is directed outward.