STRESS FIELD IN AN ELASTIC PLATE WITH MULTIPLE CRACKS

We will use Kachanov’s approximate method to analyze stresses in an elastic plate containing multiple cracks.

1. PERIODIC ARRAY OF CRACKS

In this section, we consider an infinite number of identical cracks of size $2l$, distant of $h$, periodically aligned along the $x$ axis (see Fig.1). The elastic plate is submitted to a remote tension $p^\infty$.

1. How would you apply, in this case, the principle of superposition?

2. Write the traction $p(x)$ on any given crack - centered, for the sake of simplicity, in $x = 0$ - as the sum of two terms, $p^\infty$, and a sum of tractions due to all the other cracks (centered in $x_k = hk$).

3. We consider now that each $p_k(x)$ is generated by the (yet unknown) uniform average traction $\langle p \rangle$ acting on the $k^{th}$ crack. Write the expression of $p_k(x)$ as a function of $\langle p \rangle$.

4. Take the average of traction $p_k(x)$ over the crack under consideration, centered in $x = 0$. How would you define the transmission factor $\Lambda(k)$ characterizing the attenuation of the average traction in transmission of stresses from crack $k$ onto the crack in consideration?

5. Since all the cracks are equivalent in this problem, $p(x)$ (Question 2) also has an average equal to $\langle p \rangle$. What is the expression of $\langle p \rangle$ as a function of the $\Lambda(k)$?

2. ARRAY OF PARALLEL SURFACE CRACKS

In this section, we want to model a surface with flaws. We model it as a line from which $N$ parallel cracks progress into the 2D material (see Fig. 2). The ensemble of cracks is submitted to a remote mode I loading $p^\infty$ as sketched in Fig. 2, but, because of the interactions between the cracks, each crack $i$ ($i = 1, ...N$) is submitted locally to both a normal traction $p_i(x)$ and a shear traction $t_i(x)$.

1. How to express the principle of superposition in this case?

2. The method consists in assuming that all cracks ($j \neq i$ if crack $i$ is under
consideration here) are loaded by (yet unknown) uniform average normal and shear tractions \( \langle p_j \rangle \) and \( \langle t_j \rangle \).

What are, according to you, the limitations of the method? Are there dimensionless parameters in the problem which should stay small for the calculation to remain valid?

Express the normal and shear tractions, respectively \( p_i(x) \) and \( t_i(x) \) acting on crack \( i \), as functions of the \( \langle p_j \rangle \) and \( \langle t_j \rangle \) for \( j \neq i \). We will assume that if crack \( j \) is loaded by a normal stress of unit intensity, it produces a normal stress \( \sigma_{jn}^{nn}(x) \) and a shear stress \( \sigma_{jn}^{tn}(x) \). Similarly, if crack \( j \) is loaded with a shear stress of unit intensity, it will produce a normal stress \( \sigma_{jt}^{tn}(x) \) and a shear stress \( \sigma_{jt}^{tt}(x) \).

3. One has to introduce four transmission factors: \( \Lambda_{nn}^{ji}, \Lambda_{tn}^{ji}, \Lambda_{nt}^{ji} \) and \( \Lambda_{tt}^{ji} \) to account for the attenuation of the average normal and shear tractions in transmission of stresses from crack \( j \) to crack \( i \). Take the average of the previous equations along crack \( i \), and express \( \langle \sigma_i \rangle \) and \( \langle t_i \rangle \) as a function of these transmission coefficients.

How many equations and how many unknown are there in the problem?

Knowing all the average tractions for a given configuration of cracks, we could compute the non-uniform normal and shear tractions actually acting on each crack, respectively \( p_i(x) \) and \( t_i(x) \), and hence determine the stress intensity factors \( K_I \) and \( K_{II} \) at the tip of each crack \( i \).

This approximation was used to simulate the progression of surface cracks in
Fig. 2. An array of surface cracks of various lengths separated by a distance \( h \), submitted to a remote normal traction \( p^\infty \).

Glass under stress corrosion: as a matter of fact, knowing the stress intensity factors allows to determine the velocity of all cracks under stress corrosion. It was shown that the time to failure depends crucially on cracks interactions.