

California Institute of Technology Department of Computing + Mathematical Sciences

ACM/IDS 104 Applied Linear Algebra

Fall 2025

Lectures: Tue & Thu at 10:30am-11:50am in Kerckhoff 125

Instructor: Konstantin (Kostia) Zuev

Office: Annenberg 114

Email: kostia@caltech.edu (please include "104" in the subject line)

Office Hour: Thu 2pm-3pm, or by appointment (please, send an email to schedule)

Head TA: Kieran Vlahakis (<u>kierangio@caltech.edu</u>)

TAs and OHs: https://piazza.com/caltech/fall2025/acm104/staff

You are welcome to attend as many office hours as you like.

Course Description

This is an intermediate linear algebra course aimed at a diverse group of students, including sophomores in applied and computational mathematics, computer science, and information and data sciences, as well as graduate students in science and engineering. The main goal of the course is to provide an introduction to the fundamental ideas, concepts, and methods of applied linear algebra and illustrate them with important applications. Topics covered include linear systems, vector spaces and bases, fundamental matrix subspaces, inner products, norms, k-means algorithm, positive-definite and Gram matrices, application to text classification, least squares solutions and applications to data fitting, orthogonality, the QR factorization, eigenvalues and eigenvectors, the Gerschgorin theorem, the Perron-Frobenius theorem, the PageRank algorithm, the spectral theorem, optimization principles for eigenvalues, principal component analysis, spectral method for graph partitioning, and singular value decomposition with its applications.

Prerequisites

- Ma 1 abc.
- Some familiarity with MATLAB, e.g. ACM 11, is desired.

Textbooks

- Comprehensive Lecture Notes (will be posted on Piazza after each lecture)
- P.J. Olver & C. Shakiban, *Applied Linear Algebra* (covers ~ 2/3 of the course)
- N. Johnston, Advanced Linear Algebra

Course Plan

The following is a detailed tentative outline of the topics to be covered this term.

- Linear systems, Gaussian elimination, PLU factorization
- Vector spaces, linear independence, span, basis, dimension
- Fundamental matrix subspaces, fundamental theorem of linear algebra
- Inner products, norms, Cauchy-Schwarz and triangle inequalities, normed spaces
- Application: k-means algorithm for clustering
- Positive definite matrices, Gram matrices, application to text classification
- Least square solutions, optimization of general quadratic functions
- Application of least squares to data fitting, weighted least squares, polynomial interpolation
- Orthogonality, Gram-Schmidt process, orthogonal matrices, QR factorization
- Legendre polynomials, orthogonal projections, polynomial approximation
- Eigenvalues, eigenvectors, eigenbases, Gerschgorin's theorem
- Application: PageRank algorithm for ranking websites, Perron's theorem, power method
- Diagonalization, symmetric matrices, spectral theorem, optimization interpretation of eigenvalues and eigenvectors
- Application: principal component analysis
- Application: spectral method for graph partitioning
- Singular values and singular vectors, singular value decomposition, condition number, pseudoinverse
- Matrix norms, best rank-k approximation, the Eckart-Young theorem
- Applications of SVD to image compression, political science, and facial recognition (eigenfaces)

Practice Problems

Each lecture will be accompanied by two practice problems: a somewhat easier, more practical Problem A, and a more difficult, more conceptual Problem B. The main goal of the practice problems is threefold: to help you better understand the material covered in the corresponding lecture, to help you prepare to solve problems in problem sets and exams, and to accommodate the diversity of students' math backgrounds by providing both easier and more challenging problems. These problems are for self-practice: they will not be graded, and the solutions (posted on Piazza) also illustrate the expected level of rigor for problem sets and exams.

Grading

Your final grade will be based on your total score. Your total score is a weighted average of Problem Sets (60%), Midterm exam (20%), and Final exam (20%). You can increase your total score by up to 5% if you participate actively in Piazza discussions in the Q&A section¹. Every answer submitted before TAs or instructor answer, which is later endorsed as a "good answer" by TAs or instructor, gets 1% of the total score. There are no fixed thresholds for grades, but if your total score is 90% (80%, 70%, 60%), then you are guaranteed at least "A" ("B", "C", "D").

Problem Sets

There will be six Problem Sets. Problems (and solutions) will be posted on Piazza. For assignment and due dates see "Important Dates" below. Late submissions will not be accepted, but the Problem Set with the lowest score will be dropped and not counted toward your total score. Submitting wrong files or files in a wrong format is considered as a late submission. Extensions may be granted for academic, personal, or medical reasons. For extensions, please email the Head TA.

Exams

There will be two exams: Midterm (based on Lectures 1-8) and Final (based on Lectures 9-17). The Head TA will provide a review session before each exam. Both exams are take-home, self-timed, and closed-book, but you can use one sheet (double-sided) of your own notes. You can use your electronic devices only for typing and for basic arithmetic operations.

Ethical Use of AI

You can use AI tools (e.g., ChatGPT) to support your learning in this course, but only in ethical and responsible ways. For example, it is fine to use AI to generate a practice exam based on the topics covered in the course. However, using AI to directly solve your problem sets or exams, to give you hints, or check your solutions for correctness is not allowed, as it undermines your learning and violates Caltech's Honor Code. When in doubt, ask yourself: would it be acceptable for a tutor to do this for you? If not, then it is also not appropriate to ask an AI to do it. Most importantly, keep in mind that you are here to train your own neural network, not the artificial one.

Collaboration Policy

A detailed collaboration policy is given on the course website at: http://www.its.caltech.edu/~zuev/teaching/2025Fall/CollaborationACM104.pdf

In general, collaboration is encouraged everywhere except for the exams. Let's help each other and learn together! If you get stuck on a homework problem, I encourage you to discuss it with other students (offline or online on Piazza). But remember that you will have to prepare and submit your solution by yourself. No collaboration is allowed on the exams.

Important Dates (All times are Pacific Times)

	Available	Due
Problem Set 1	1pm Tue, Oct 07	9pm Tue, Oct 14
Problem Set 2	1pm Tue, Oct 14	9pm Tue, Oct 21
Problem Set 3	1pm Tue, Oct 21	9pm Tue, Oct 28
Head TA Review	10:30am Tue, Oct 28	
Midterm Exam	1pm Tue, Oct 28	9pm Tue, Nov 04
Problem Set 4	1pm Tue, Nov 04	9pm Tue, Nov 11
Problem Set 5	1pm Tue, Nov 11	9pm Tue, Nov 18
Problem Set 6	1pm Tue, Nov 18	9pm Tue, Nov 25
Head TA Review	10:30am Thu, Dec 04	
Final Exam	1pm Thu, Dec 04	9pm Thu, Dec 11

¹ If you are interested in being a TA next year, try to be active on Piazza and help other students by answering their questions.

Websites

- Course website:
 - http://www.its.caltech.edu/~zuev/teaching/2025Fall/ACM104.htm
- Lecture notes, practice problems, problem sets, exams, solutions, announcements, and class discussions will be managed via Piazza, which is designed such that you can get a quick help from your classmates, TA(s), and instructor. Instead of emailing questions to the teaching staff, I encourage you to post your questions on Piazza because
 - O You will get the answers faster
 - O Your classmates may also benefit from seeing the answers to your questions.
- Here is the Piazza page:
 - http://www.piazza.com/caltech/fall2025/acm104/home
- Problem sets and exams will be graded via Gradescope.
 - o If you are a **registered student**, you will be enrolled on Gradescope by the end of the 1st week of classes, and you will receive a notification from Gradescope about your enrollment.
 - Please make sure that the email that you use on Gradescope is your official Caltech email.
 - o If you are a **registered student** but have not been enrolled on Gradescope by the end of the 1st week of classes, please email the Head TA as soon as possible and ask to enroll you to Gradescope. Your absence on Gradescope means that, according to my records, you are not registered for the course.
 - o If you want just to **audit the course**, it is fine, you will have access to Piazza and all course materials there (please email me and I will enroll you on Piazza), but you will not have access to Gradescope and your submissions will not be graded. If you audit the course this year, you should not register for the course in the future.

To submit your solution via Gradescope, you need to create a single PDF (not images) that contains the whole solution (for example, by scanning your solution), and then upload it to Gradescope. Here is a useful link:

How can I submit my homework as a PDF?
 https://guides.gradescope.com/hc/en-us/articles/21862105254413-How-can-I-submit-my-homework-as-a-PDF

Should you have any questions regarding Gradescope, please ask on Piazza: we will have many experts there.

Suggested Study Process

To get the most out of ACM 104, here is my suggested study process²:

- Have Enough Sleep: Good sleep is an important prerequisite for learning.
- Attend Lectures: Focus on understanding the big picture of what is going on.
- Review Lecture Notes: Ideally on the same day they are released, make sure everything is clear.
- Ask and Answer Questions: If something is not clear, ask on Piazza; help your classmates by answering their questions.
- Summarize in Your Own Notes: After each lecture, very briefly summarize my notes, extract the essence.
- Work on Practice Problems: Attempt to solve the practice problems and review my solutions.
- Attend Office Hours: Interact with the instructor, TAs, and other students.
- Start Early: Begin each problem set on the day it is released (or as soon as possible after that).
- Finish Early: Aim to complete each problem set and exam at least one day before the deadline.
- Stuck? Ask for Help: If you get stuck on a problem set problem, ask for hints on Piazza

Keep in Mind

My goal is to help you understand and learn the material. Understanding is a creative process that takes time and effort. If you do not understand something, please ask me. If you are struggling to balance the workload, talk to me. If you have any concerns, let me know. Keep in mind that I am here to help.

Honor Code

"No member of the Caltech community shall take unfair advantage of any other member of the Caltech community."

² Based on Stanislav Dehaene, How We Learn: The New Science of Education and the Brain. 2020: Penguin Books Limited.