

Quiz 7

Name: Super Student

USC ID: _____

Problem 1 (3 points)

Find the derivative of the following function:

$$f(x) = \ln(e^x + e^{-x})$$

$$f'(x) = \frac{1}{e^x + e^{-x}} \cdot (e^x + e^{-x})' = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Problem 2 (3 points)

Use logarithmic differentiation to find the derivative of the following function:

$$f(x) = x^2 e^{-x} (x+1)^2$$

$$f'(x) = f(x) \cdot (\ln f(x))'$$

$$\bullet \ln f(x) = \ln(x^2 e^{-x} (x+1)^2) = \ln x^2 + \ln e^{-x} + \ln (x+1)^2 = 2 \ln x - x + 2 \ln(x+1)$$

$$\bullet (\ln f(x))' = \frac{2}{x} - 1 + \frac{2}{x+1}$$

$$\bullet f'(x) = x^2 e^{-x} (x+1)^2 \cdot \left(\frac{2}{x} - 1 + \frac{2}{x+1} \right)$$

Problem 3 (4 points, Final Exam Fall 2010)

Find the derivative of the following function:

$$f(x) = (x^2 e^x)^x$$

Let us use logarithmic differentiation : $f'(x) = f(x) (\ln f(x))'$

$$\bullet \ln f(x) = \ln((x^2 e^x)^x) = x \ln(x^2 e^x) = x (\ln x^2 + \ln e^x) = \\ = x (2 \ln x + x) = 2x \ln x + x^2$$

$$\bullet (\ln f(x))' = 2 \ln x + 2 + 2x = 2(1 + x + \ln x)$$

$$\bullet f(x) = (x^2 e^x)^x \cdot (1 + x + \ln x) \cdot 2$$