Quiz 6

Name: Super Student

USC ID: ________________

Problem 1 (3 points)
Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval:
$f(x) = x^2 - 4x + 1, \quad 0 \le x \le 3$

1) $f'(x) = 2x - 4$

$f'(x) = 0 \iff \underline{x = 2}$

So $x = 2$ is the only critical number in $(0,3)$

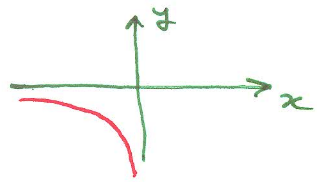
2) Compute $f(x)$ at the critical point and at the endpoints:

$f(0) = \underline{1}$ absolute max $(0,1)$

$f(2) = 4 - 8 + 1 = \underline{-3}$ absolute min $(2,-3)$

$f(3) = 9 - 12 + 1 = -2$

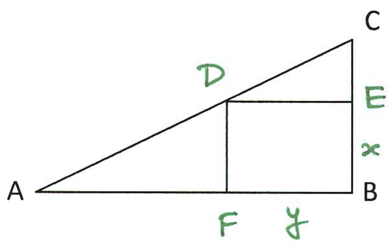
Problem 2 (3 points)
Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval:
$f(x) = \frac{1}{x}, \quad x < 0$



$f'(x) = -\frac{1}{x^2} < 0 \Rightarrow f(x)$ is decreasing on $(-\infty, 0)$

$\Rightarrow$ there is no critical numbers in $(-\infty, 0)$ and there is no endpoints $\Big\} \Rightarrow$ no abs max, no abs min

Problem 3 (4 points) A rectangle is inscribed in a right triangle, as shown in the figure. AB=4, BC=3, AC=5. Find the dimensions of the inscribed rectangle with the largest area.



① Let $EB = x$, $FB = y$

Then, the area of FDEB: $\boxed{A = x \cdot y}$

By similar triangles:

$\frac{DE}{AB} = \frac{CE}{CB} \Rightarrow \frac{y}{4} = \frac{3-x}{3} \Rightarrow \boxed{y = \frac{4}{3}(3-x)}$

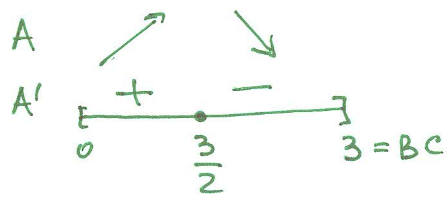
② Thus, we want

$\boxed{\begin{array}{l} A = x \cdot y \to \max \\ y = \frac{4}{3}(3-x) \\ x > 0,\ x < BC = 3 \\ y > 0,\ y < AB = 4 \end{array}}$

$A = \frac{4}{3}x(3-x)$

$A' = \frac{4}{3}(3-2x)$

$A' = 0 \iff \boxed{x = \frac{3}{2}}$



$\Rightarrow x = \boxed{\frac{3}{2}}$ gives the largest area

$y = \frac{4}{3}(3 - 1.5) = \boxed{2}$