

Quiz 5

Name: Super Student

USC ID: _____

Problem 1 (3 points)

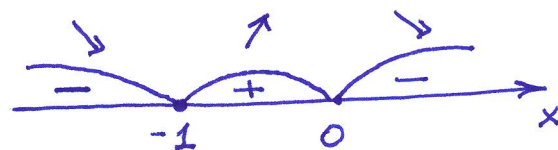
Find intervals of increase and decrease for the given function:

$$f(x) = \frac{x^2 + 2x + 1}{x^2}$$

$$1. \quad f'(x) = \frac{(2x+2)x^2 - 2x(x^2+2x+1)}{x^4} = \frac{2x^3 + 2x^2 - 2x^3 - 4x^2 - 2x}{x^4} = -2 \frac{x+1}{x^3}$$

$$f'(x) = 0 \Leftrightarrow \boxed{x = -1}, \quad f'(x) \text{ does not exist} \Leftrightarrow \boxed{x = 0}$$

2. a) if $x < -1 \Rightarrow f' < 0$ ($f'(-2) < 0$)
 b) if $-1 < x < 0 \Rightarrow f' > 0$ ($f'(-1/2) > 0$)
 c) if $x > 0 \Rightarrow f' < 0$ ($f'(1) < 0$)



Intervals of increase: $(-1, 0)$
 Intervals of decrease: $(-\infty, -1), (0, +\infty)$

Problem 2 (3 points)

For the given function, find its critical numbers, and classify them (rel. max, rel. min., or neither)

$$f(x) = \frac{x^2 + 2x + 1}{x^2}$$

$f'(x)$ equals to zero or does not exist for $x = -1, x = 0$

But $x = 0$ is not in the domain of $f(x)$.

Therefore, $\boxed{x = -1}$ is the only critical number of $f(x)$

Since $f'(x) < 0$ to the left of $x = -1$, and $f'(x) > 0$ to the right of $x = -1$,

$\boxed{x = -1 \text{ is a relative minimum}}$

Problem 3 (4 points)

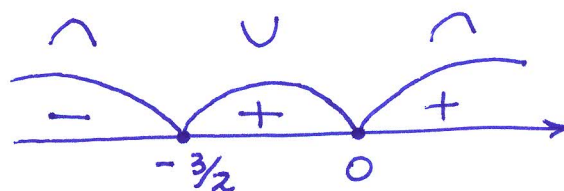
Determine where the graph of the given function is concave upward and concave downward

$$f(x) = \frac{x^2 + 2x + 1}{x^2}$$

$$1. \quad f''(x) = \left(-2 \frac{x+1}{x^3}\right)' = -2 \frac{x^3 - 3x^2(x+1)}{x^6} = -2 \frac{x - 3(x+1)}{x^4} = +2 \frac{2x+3}{x^4}$$

$$f''(x) = 0 \Leftrightarrow \boxed{x = -\frac{3}{2}}, \quad f''(x) \text{ does not exist} \Leftrightarrow \boxed{x = 0}$$

2. a) $x < -\frac{3}{2} \Rightarrow f''(x) < 0$
 b) $-\frac{3}{2} < x < 0 \Rightarrow f''(x) > 0$



- c) $x > 0 \Rightarrow f''(x) > 0$

The graph is concave upward: $(-\frac{3}{2}, 0), (0, +\infty)$

The graph is concave downward: $(-\infty, -\frac{3}{2})$