Quiz 4

Name: Super Student

USC ID: _____

Problem 1 (3 points)
Find the derivative of the given function: $f(x) = (x+1)^{1.001}$

Using the chain rule, we have:

$$f'(x) = 1.001\,(x+1)^{0.001} \cdot (x+1)' = 1.001\,(x+1)^{0.001}$$

Problem 2 (3 points)
Find the derivative of the given function: $f(x) = \frac{x+5}{\sqrt{1+x^2}}$

$$f'(x) = \frac{(x+5)'\sqrt{1+x^2} - (x+5)(\sqrt{1+x^2})'}{1+x^2} = \frac{\sqrt{1+x^2} - (x+5)\frac{1}{2}\cdot\frac{1}{\sqrt{1+x^2}}\cdot 2x}{1+x^2} =$$

$$= \frac{1+x^2 - x(x+5)}{(1+x^2)^{3/2}} = \frac{1-5x}{(1+x^2)^{3/2}}$$

Problem 3 (4 points, this problem was in the final exam in 2010)
Find an equation of the tangent line to $x^2 + 2y^3 = \frac{3}{xy}$ at the point (1,1).

Let us use the point-slope form of a line equation:

$y - y_0 = m(x - x_0)$ In our case $x_0 = 1$, $y_0 = 1$

The slope $m = \left.\frac{dy}{dx}\right|_{\substack{x=x_0 \\ y=y_0}}$

Using implicit differentiation: $2x + 6y^2y' = \frac{-3(xy)'}{(xy)^2} = \frac{-3(y+xy')}{x^2y^2}$

$$2x^3y^2 + 6x^2y^4y' = -3y - 3xy'$$

$$y'(6x^2y^4 + 3x) = -3y - 2x^3y^2$$

$$y' = -\frac{3y + 2x^3y^2}{6x^2y^4 + 3x}$$

So, $\left.\frac{dy}{dx}\right|_{\substack{x=1 \\ y=1}} = -\frac{3+2}{6+3} = -\frac{5}{9}$

Finally, we have

$$\boxed{y - 1 = -\frac{5}{9}(x-1)}$$