

Quiz 3

Name: Super Student

USC ID: _____

Problem 1 (3 points)

Find the derivative of the given function

$$f(x) = \frac{2x^2 - 3x + 4}{x^4 + 1}$$

$$\begin{aligned} f'(x) &= \frac{(2x^2 - 3x + 4)'(x^4 + 1) - (2x^2 - 3x + 4)(x^4 + 1)'}{(x^4 + 1)^2} = \frac{(4x - 3)(x^4 + 1) - 4x^3(2x^2 - 3x + 4)}{(x^4 + 1)^2} \\ &= \frac{4x^5 - 3x^4 + 4x - 3 - 8x^5 + 12x^4 - 16x^3}{(x^4 + 1)^2} = \frac{-4x^5 + 9x^4 - 16x^3 + 4x - 3}{(x^4 + 1)^2} \end{aligned}$$

Problem 2 (3 points)

Find all points on the the graph of the given function where the tangent line has slope $m=3$

$$f(x) = \sqrt[3]{x^3}$$

Slope of the tangent line at the point $(x, f(x))$ is given by $f'(x)$

$$f'(x) = \left(x^{\frac{3}{2}}\right)' = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2} \sqrt{x}$$

Therefore, we need to solve $\frac{3}{2} \sqrt{x} = m = 3$ for x .

$$\frac{3}{2} \sqrt{x} = 3 \Leftrightarrow \boxed{x=4}, \boxed{f(4)=8}. \text{ Final answer: } \underline{\underline{\text{point } (4, 8)}}$$

Problem 3 (4 points)

Find the second derivative of the given function

$$f(x) = \sqrt[3]{x^2} + \sqrt[2]{x^3}$$

$$1. f'(x) = \left(x^{\frac{2}{3}} + x^{\frac{3}{2}}\right)' = \frac{2}{3} x^{-\frac{1}{3}} + \frac{3}{2} x^{\frac{1}{2}}$$

$$2. f''(x) = \left(\frac{2}{3} x^{-\frac{1}{3}} + \frac{3}{2} x^{\frac{1}{2}}\right)' = \boxed{-\frac{2}{9} x^{-\frac{4}{3}} + \frac{3}{4} x^{-\frac{1}{2}}}$$