Quiz 2

Name: Super Student

USC ID: ____________

Problem 1 (3 points)
Calculate the following limit. If the limit is infinite, indicate whether it is +∞ or −∞.

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-3x+2}$$

Since $x^2+x-6=(x-2)(x+3)$ and $x^2-3x+2=(x-2)(x-1)$

$$\lim_{x\to 2}\frac{x^2+x-6}{x^2-3x+2}=\lim_{x\to 2}\frac{(x-2)(x+3)}{(x-2)(x-1)}=\lim_{x\to 2}\frac{x+3}{x-1}=\frac{5}{1}=\boxed{5}$$

Problem 2 (3 points)
Calculate the following one-sided limit. If the limit is infinite, indicate whether it is +∞ or −∞.

$$\lim_{x\to 2+}\sqrt{\frac{x+3}{3x-6}}$$

When $x\to 2+$, the nominator $\sqrt{x+3}\to\sqrt{5}+$, but the denominator $\sqrt{3x-6}\to 0+$. Therefore, $\lim_{x\to 2+}\sqrt{\frac{x+3}{3x-6}}=\boxed{+\infty}$

Problem 3 (4 points) Suppose that $f(x)=\begin{cases} x^2+1, & if\ x\geq 2 \\ \frac{x-7}{x-3}, & if\ x<2 \end{cases}$

Decide if this function is continuous at $x=2$.

The function is continuous at $x=2$ $\Longleftrightarrow$
1) $f(2)$ exists
2) $\lim_{x\to 2} f(x)$ exists
3) $\lim_{x\to 2} f(x)=f(2)$

Let us check all these conditions.

1) $f(2)=2^2+1=5$ ($f(2)$ is defined)

2) $\left.\begin{aligned}\lim_{x\to 2-} f(x)&=\lim_{x\to 2-}\frac{x-7}{x-3}=\frac{2-7}{2-3}=\underline{5}\\ \lim_{x\to 2+} f(x)&=\lim_{x\to 2+}(x^2+1)=2^2+1=\underline{5}\end{aligned}\right\}\Rightarrow \lim_{x\to 2} f(x)=5$

So $\lim_{x\to 2} f(x)$ exists, and, moreover,

3) $\lim_{x\to 2} f(x)=f(2)=5$

Thus, the function is continuous at $x=2$