

Quiz 12

Name: Super Student

USC ID: _____

Problem 1 (5 points)

Evaluate the following double integral

$$\begin{aligned}\int_0^2 \int_{-1}^1 x^2 e^{xy} dx dy &= \int_{-1}^1 \left(\int_0^2 x^2 e^{xy} dy \right) dx = \int_{-1}^1 \left(x^2 \int_0^2 e^{xy} dy \right) dx = \int_{-1}^1 x^2 \left. \frac{e^{xy}}{x} \right|_0^2 dx \\&= \int_{-1}^1 x (e^{2x} - 1) dx = \int_{-1}^1 x e^{2x} dx - \int_{-1}^1 x dx = \frac{1}{2} \int_{-1}^1 x d e^{2x} - \left. \frac{x^2}{2} \right|_{-1}^1 \\&= \frac{1}{2} \left(x e^{2x} \Big|_{-1}^1 - \int_{-1}^1 e^{2x} dx \right) - \left(\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2} \left(e^2 + e^{-2} - \frac{1}{2} e^{2x} \Big|_{-1}^1 \right) \\&= \frac{1}{2} \left(e^2 + e^{-2} - \frac{1}{2} e^2 + \frac{1}{2} e^{-2} \right) = \frac{1}{2} \left(\frac{1}{2} e^2 + \frac{3}{2} e^{-2} \right) = \frac{1}{4} (e^2 + 3e^{-2})\end{aligned}$$

Problem 2 (5 points)

Evaluate the following double integral

$$\begin{aligned}\int_1^e \int_1^e \ln(xy) dy dx &= \int_1^e \left(\int_1^e \ln x + \ln y dy \right) dx = \int_1^e \left(\int_1^e \ln x dy + \int_1^e \ln y dy \right) dx \\&= \int_1^e \left(e \ln x + y \ln y \Big|_1^e - \int_1^e y d \ln y \right) dx = \\&= \int_1^e \left(e \ln x + e - \int_1^e dy \right) dx = \int_1^e (e \ln x + e - e + 1) dx \\&= e \int_1^e \ln x dx + \int_1^e dx = e \left(x \cdot \ln x \Big|_1^e - \int_1^e x d \ln x \right) + e - 1 \\&= e \left(e - \int_1^e dx \right) + e - 1 = e(e - e + 1) + e - 1 = 2e - 1\end{aligned}$$