Quiz 11

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Problem 1 (5 points)

Use the chain rule to find $\frac{dz}{dt}$, where $z(x,y) = \ln(xy)$, $x = t$, $y = \ln t$.

$$\frac{dz}{dt} = \left.\frac{\partial z}{\partial x}\right|_{\substack{x=x(t)\\ y=y(t)}} \cdot \frac{dx}{dt} + \left.\frac{\partial z}{\partial y}\right|_{\substack{x=x(t)\\ y=y(t)}} \cdot \frac{dy}{dt} \qquad \begin{cases} \frac{dx}{dt} = 1 \\ \frac{dy}{dt} = \frac{1}{t} \end{cases}$$

$$\frac{\partial z}{\partial x} = \frac{1}{xy} \cdot (xy)'_x = \frac{y}{xy} = \frac{1}{x} \Rightarrow \left.\frac{\partial z}{\partial x}\right|_{\substack{x=x(t)\\ y=y(t)}} = \frac{1}{t}$$

$$\frac{\partial z}{\partial y} = \frac{1}{xy}(xy)'_y = \frac{x}{xy} = \frac{1}{y} \Rightarrow \left.\frac{\partial z}{\partial y}\right|_{\substack{x=x(t)\\ y=y(t)}} = \frac{1}{\ln t}$$

Thus, $\boxed{\frac{dz}{dt} = \frac{1}{t} + \frac{1}{t \ln t}}$

Problem 2 (5 points)

Find all critical points of the given function and classify each as a relative minimum, a relative maximum, or a saddle point. $f(x,y) = (x^2+1)(y^2-1)$

1. Critical points

$$\begin{cases} f_x = 2x(y^2-1) = 0 \\ f_y = 2y(x^2+1) = 0 \end{cases}$$

since $x^2+1 > 0$, $2y(x^2+1) = 0 \Leftrightarrow \underline{\underline{y=0}}$

Therefore, from the first equation: $2x(0-1) = 0 \Rightarrow \underline{\underline{x=0}}$

$(0,0)$ is the critical point.

2. $D(x,y) = f_{xx} f_{yy} - (f_{xy})^2$ $\qquad f_{xx} = 2(y^2-1)$ $\qquad f_{xy} = 4xy$

$D(x,y) = 4(y^2-1)(x^2+1) - (4xy)^2$ $\qquad f_{yy} = 2(x^2+1)$

3. $D(0,0) = 4(-1)(1) - 0 = -4 < 0$

$\Rightarrow (0,0)$ is a saddle point