Problem 1. Find the absolute maximum and absolute minimum (write "none" if there are none) of the given function on the specified interval.

$(a)\ f(x) = -x^2 + 4x + 7; \quad 0 \le x \le 3$

1.) $f'(x) = -2x+4$

$f'(x) = 0 \iff x = 2$

Thus, $x=2$ is the critical point

2.) $f(2) = -4+8+7 = 11$

$f(0) = 7$

$f(3) = -9+12+7 = 10$

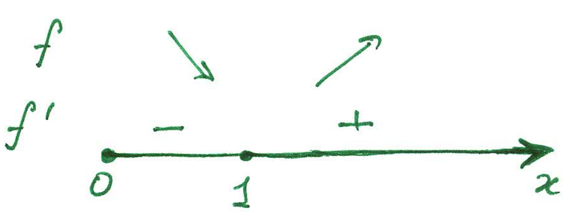
(a)
Abs. Max: x= 2 y= 11
Abs. Min: x= 0 y= 7

$(b)\ f(x) = x - \ln x; \quad x > 0$

$f'(x) = 1 - \frac{1}{x}$

$f'(x) = 0 \iff 1 = \frac{1}{x} \iff x = 1$

Thus $x = 1$ is the only critical point



$\Rightarrow$ $x = 1$ is the abs. min, $f(1) = 1$

and there is no abs. max

(b)
Abs. Max: x= NA y= NA
Abs. Min: x= 1 y= 1

Problem 2. An LA Galaxy team store can obtain soccer balls with Beckham's signature at a cost of $10 per ball. The store has been offering the balls at $20 apiece and, at this price, has been selling 30 balls per month. The store is planning to lower the price to stimulate sales and estimates that for each dollar reduction in the price, 5 more balls will be sold each month. At what price should the balls be sold in order to maximize the total monthly profit?

$$c = 10 \ \frac{\$}{\text{ball}}$$

$$c_1 = 20 \ \frac{\$}{\text{ball}} \implies n_1 = 30 \ \frac{\text{balls}}{\text{month}}$$

Let the reduction in the price be x \$/ball

Then

$$c_2 = (20 - x) \ \frac{\$}{\text{ball}} \implies n_2 = (30 + 5x) \ \frac{\text{balls}}{\text{month}}$$

Monthly profit : $p = (c_2 - c) \cdot n_2$

$$p = (20 - x - 10)(30 + 5x) =$$

$$= (10 - x)(30 + 5x) = 300 - 30x + 50x - 5x^2$$

We want $p(x) \to \max$

$$= -5x^2 + 20x + 300.$$

$$p'(x) = -10x + 20$$

$$p'(x) = 0 \iff \underline{\underline{x = 2}}$$

p $\nearrow$ $\searrow$

p' $+$ $-$

2

$\implies$ $x = 2$ is abs max.

$$\implies c_2 = 18 \ \frac{\$}{\text{ball}}$$

$$\boxed{\text{Price} = 18 \ \frac{\$}{\text{ball}}}$$

Problem 3. Find the derivatives of the following functions:

$(a)\ f(x) = \ln((1+x^3)^x) = x \ln(1+x^3)$

$$f'(x) = \left(x \cdot \ln(1+x^3)\right)' = \ln(1+x^3) + x \cdot \frac{1}{1+x^3}(1+x^3)'$$

$$= \ln(1+x^3) + \frac{3x^3}{1+x^3}$$

(a)
$\boxed{\ln(1+x^3) + \frac{3x^3}{1+x^3}}$

$(b)\ f(x) = x^{10x}$

$$f'(x) = f(x) \cdot (\ln f(x))'$$

$$\ln f(x) = \ln x^{10x} = 10x \ln x$$

$$(\ln f(x))' = 10 \ln x + 10x \cdot \frac{1}{x} = 10(\ln x + 1)$$

$$f'(x) = x^{10x} \cdot 10 \cdot (\ln x + 1)$$

(b)
$\boxed{10x^{10x}(\ln x + 1)}$

Problem 4. After the LA Galaxy team store had reduced the price for soccer balls with Beckham's signature, the total number of sold balls $N(t)$ started to grow exponentially, $N(t) = N_0 e^{kt}$, where N_0 and k are constants, and t is the number of months (t=0 corresponds to the end of October, t=1 corresponds to the end of November, etc). If 40 balls had been sold by the end of October, and 60 balls had been sold by the end of November, how many will have been sold by the end of December?

$$N(t) = N_0 e^{kt}$$

$$\begin{cases} N(0) = 40 \\ N(1) = 60 \end{cases} \Rightarrow \begin{cases} N_0 = 40 \\ N_0 e^{k} = 60 \end{cases} \Rightarrow e^{k} = \frac{3}{2}$$

We need to find $N(2)$

$$N(2) = N_0 e^{2k} = N_0 \cdot (e^k)^2 = 40 \cdot \left(\frac{3}{2}\right)^2 = 40 \cdot \frac{9}{4} = 90$$

Number of balls= 90

Problem 5. Evaluate the following indefinite integrals

(a) $\int \frac{x-1}{\sqrt{x}}dx = \int x^{\frac{1}{2}} - x^{-\frac{1}{2}} dx = \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + C$

(a) $\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + C$

(b) $\int \frac{2x\ln(x^2+1)}{x^2+1}dx = \begin{bmatrix} u = x^2+1 \\ du = 2x\,dx \end{bmatrix} = \int \frac{\ln u}{u} du = \begin{bmatrix} v = \ln u \\ dv = \frac{du}{u} \end{bmatrix}$

$= \int v\,dv = \frac{v^2}{2} + C = \frac{1}{2}(\ln u)^2 + C$

$= \frac{1}{2}\left(\ln(x^2+1)\right)^2 + C$

Or $\int \frac{2x\ln(x^2+1)}{x^2+1}dx = \begin{bmatrix} u = \ln(x^2+1) \\ du = \frac{2x\,dx}{x^2+1} \end{bmatrix} = \int u\,du = \frac{u^2}{2} + C$

$= \frac{1}{2}\left(\ln(x^2+1)\right)^2 + C$

(b) $\frac{1}{2}\left(\ln(x^2+1)\right)^2 + C$