Horseracing Simulation Algorithm
for evaluation of small failure probabilities

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Basic Idea

Horseracing Simulation scheme

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  ▶ Construction of the empirical CDF and its updating
  ▶ Stopping criterion

Horseracing Simulation Algorithm

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  ▶ Simulation results
Basic Idea

Reliability Problem:

\[ \Omega = \int_\Omega \pi_0(x) \, dx \]

Failure domain:

\[ \Omega = \{ x \in \mathbb{R}^N \mid g(x) > z^* \} \]

Limit-state function:

\[ g: \mathbb{R}^N \rightarrow \mathbb{R}^+ \]

\[ z^* = g(x) \]

\[ F(z) = \left( 1 - p \right) \frac{\partial z}{\partial F(z)} \]

Basic Idea:

to approximate the CDF \( F \) in the neighbourhood of \( z^* \)
Basic Idea

Reliability Problem: to compute the failure probability

\[ p_\Omega = \int_\Omega \pi_0(x) dx \]
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Reliability Problem: to compute the failure probability

\[ p_\Omega = \int_\Omega \pi_0(x) \, dx \]

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Limit-state function: \( g : \mathbb{R}^N \rightarrow \mathbb{R}_+ \), \( z = g(x) \)

\[ p_\Omega = 1 - F(z^*) \]
Basic Idea

Reliability Problem: to compute the failure probability
\[ p_\Omega = \int_{\Omega} \pi_0(x) \, dx \]

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Limit-state function: \( g : \mathbb{R}^N \rightarrow \mathbb{R}_+, \; z = g(x) \)

Basic Idea: to approximate the CDF \( F \) in the neighbourhood of \( z^* \)
Basic Idea

If $z^*$ is not very far from the median $\tilde{z}$, then we can use Monte Carlo Simulation.
Basic Idea

Assume, we can propagate Monte Carlo samples towards the neighbourhood of $z^*$:
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Assume, we can propagate Monte Carlo samples towards the neighbourhood of $z^*$:

PDF of $z^{(k)}$:

$$f_k(z) = \frac{(-1)^k}{k!} f(z) \left[ \log(1 - F(z)) \right]^k$$
Basic Idea

Assume, we can propagate Monte Carlo samples towards the neighbourhood of \( z^* \):

PDF of \( z^{(k)} \):

\[
f_k(z) = (-1)^k \frac{k!}{k^k} f(z) \left[ \log(1 - F(z)) \right]^k
\]

Importance Sampling theory:

if \( z_1^{(k)}, \ldots, z_n^{(k)} \sim f_k \), then \( (z_1^{(k)}, w_1^{(k)}), \ldots, (z_n^{(k)}, w_n^{(k)}) \sim f \), where

\[
w_i^{(k)} \propto \frac{f(z_i^{(k)})}{f_k(z_i^{(k)})} \propto \frac{1}{\left[ \log(1 - F(z_i^{(k)})) \right]^k}
\]
Basic Idea

**Horseracing Simulation Scheme**

I. Sample $z_{1}^{(0)}, \ldots, z_{n}^{(0)}$ from $f_{0} = f$,
Set $k = 0$.

II. Construct the empirical CDF $F^{(k)}$ based on $\{z_{i}^{(k)}\}_{i=1}^{n}$.

While the stopping criterion $C(z^{*})$ is not fulfilled do:

III. Sample $z_{i}^{(k+1)}$ from $f_{0}(z|z \geq z_{i}^{(k)})$ for each $i = 1 \ldots n$.

IV. Construct the empirical CDF $G^{(k+1)}$ based on $\{(z_{i}^{(k+1)}, w_{i}^{(k+1)})\}_{i=1}^{n}$.

V. Update CDF $F^{(k)}$ to $F^{(k+1)}$, $(F^{(k)}, G^{(k+1)}) \leadsto F^{(k+1)}$,
Set $k = k + 1$. 

In some neighbourhood of $z^{*}$:

$F^{(k)} \approx F_{L}$
Basic Idea

Horseracing Simulation Scheme

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V. Update CDF \( F^{(k)} \) to \( F^{(k+1)} \), \( (F^{(k)}, G^{(k+1)}) \sim F^{(k+1)} \),
   Set \( k = k+1 \).

In some neighbourhood of \( z^* \), \( F^{(k)} \approx F \)
Implementation issues: Sampling

Step I. Sample from $f_0$ is easy:

- $x_1(0),...,x_n(0) \sim \pi_0$ Monte Carlo samples
- $z_1(0) = g(x_1(0)), ..., z_n(0) = g(x_n(0)) \sim f_0$

Step III. Given $z_i(k)$, sample $z_i(k+1)$ from $f_0(z|z \geq z_i(k))$
Implementation issues: Sampling

Step I. Sample from $f_0$ is easy:

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Monte Carlo samples

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\[ \ldots \]
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  \ldots \\
  z_n^{(0)} &= g(x_n^{(0)})
\end{align*} \sim f_0 \]

Monte Carlo samples

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Step I. Sample from $f_0$ is easy:

$$x_1^{(0)}, \ldots, x_n^{(0)} \sim \pi_0$$

Monte Carlo samples

$$\Rightarrow \quad \ldots \quad \sim f_0$$

$$z_1^{(0)} = g(x_1^{(0)})$$

$$z_n^{(0)} = g(x_n^{(0)})$$

Step III. Given $z_i^{(k)}$, sample $z_i^{(k+1)}$ from $f_0(z | z \geq z_i^{(k)})$
Implementation issues: Construction of the empirical CDF

Step II. Construct a zeroth approximation $F(0)$ based on $z(0)_1, \ldots, z(0)_n$

$$F(0)(z) = \frac{1}{2n} \left( \frac{2i - 1}{2i + 1} z(0)_{i+1} - \frac{2i + 1}{2i - 1} z(0)_i \right) + \left( 2i - 1 \right) z(0)_i + \left( 2i + 1 \right) z(0)_{i+1} - \frac{4i^2 + 1}{2n} \left( z(0)_{i+1} - z(0)_i \right), \quad z \in [z(0)_i, z(0)_{i+1}]$$
Implementation issues: Construction of the empirical CDF

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\[
F^{(0)}(z) = \frac{z}{n(z_{i+1}^{(0)} - z_i^{(0)})} + \frac{(2i - 1)z_{i+1}^{(0)} - (2i + 1)z_i^{(0)}}{2n(z_{i+1}^{(0)} - z_i^{(0)})}, \text{ for } z \in [z_i^{(0)}, z_{i+1}^{(0)}]
\]
Implementation issues: Construction of the empirical CDF

Step IV. Construct the empirical CDF $G^{(k+1)}$ based on the weighted samples $\{(z_i^{(k+1)}, w_i^{(k+1)})\}_{i=1}^{n}$

For $z \in [z_i^{(k+1)}, z_{i+1}^{(k+1)}]$:

$$
G^{(k+1)}(z) = \frac{w_i^{(k+1)} + w_{i+1}^{(k+1)}}{2 \left( z_{i+1}^{(k+1)} - z_i^{(k+1)} \right)} z + \\
\frac{\left( 2 \sum_{j=1}^{i} w_j^{(k+1)} - w_i^{(k+1)} \right) z_{i+1}^{(k+1)} - \left( 2 \sum_{j=1}^{i} w_j^{(k+1)} + w_{i+1}^{(k+1)} \right) z_i^{(k+1)}}{2 \left( z_{i+1}^{(k+1)} - z_i^{(k+1)} \right)}
$$
 Implementation issues: Updating of the empirical CDF

Step V. Update the CDF \( F^{(k)} \), using new information provided by \( G^{(k+1)} \), and construct a new approximation \( F^{(k+1)} \) of the CDF \( F \).
Implementation issues: Updating of the empirical CDF

Step V. Update the CDF $F^{(k)}$, using new information provided by $G^{(k+1)}$, and construct a new approximation $F^{(k+1)}$ of the CDF $F$. 

$$
F^{(k)}(z) = \begin{cases} 
F^{(k)}(z), & \text{for } z \in [z^{(0)}, z^{(1)}] \\
0, & \text{for } z \in [z^{(0)}, z^{(1)}] \\
G^{(k+1)}(z), & \text{for } z \in [z^{(k+1)}, z^{(k)}].
\end{cases}
$$
Implementation issues: Updating of the empirical CDF

Step V. Update the CDF $F^{(k)}$, using new information provided by $G^{(k+1)}$, and construct a new approximation $F^{(k+1)}$ of the CDF $F$.

$$F^{(k+1)}(z) = \begin{cases} 
F^{(k)}(z), & \text{for } z \in [z_1^{(0)}, z_1^{(k+1)}); \\
\frac{(k+1)F^{(k)}(z)+G^{(k+1)}(z)}{k+2}, & \text{for } z \in [z_1^{(k+1)}, z_n^{(0)}]; \\
\frac{kF^{(k)}(z)+G^{(k+1)}(z)}{k+1}, & \text{for } z \in (z_n^{(0)}, z_n^{(1)}]; \\
\ldots, & \text{for } z \in (z_n^{(k-1)}, z_n^{(k)}]; \\
\frac{F^{(k)}(z)+G^{(k+1)}(z)}{2}, & \text{for } z \in (z_n^{(k)}, z_n^{(k+1)}]. 
\end{cases}$$
Implementation issues: Stopping Criterion

\[ z^* : \text{the race is over when } r\% \text{ of horses reach } z^* \]

\[ \chi(k) z^* \geq r\% \]

For a target probability level of $10^{-2}$ to $10^{-5}$, choosing $r = 10\%$ is found to yield good efficiency.
Implementation issues: Stopping Criterion

$C(z^*):$ the race is over when $r\%$ of horses reach $z^*$,

\[ \chi_{z^*}^{(k)} \geq r\% \]
Implementation issues: Stopping Criterion

$C(z^*)$: the race is over when $r\%$ of horses reach $z^*$,

\[ X_{z^*}^{(k)} \geq r\% \]

For a target probability level of $10^{-2}$ to $10^{-5}$, choosing $r = 10\%$ is found to yield good efficiency.
Horseracing Simulation Algorithm

Let \( x_1^{(0)}, \ldots, x_n^{(0)} \)

be the initial samples.

Then, we calculate the limit-state function \( g \) at these samples.

If \( g(z^*) = 0 \) for some \( z^* \), then we set \( k = 0 \) and begin the algorithm.

Otherwise, we update the failure probability estimate to

\[ p_\Omega = 1 - \hat{F}^{(k)}(z^*) \]

and set \( k = k + 1 \).

Next, we sample \( z_1^{(k)}, \ldots, z_n^{(k)} \)

and \( w_1^{(k+1)}, \ldots, w_n^{(k+1)} \)

and use them in the updated algorithm to get \( \hat{F}^{(k+1)} \) and \( G^{(k+1)} \).

If \( \chi^{(k)}_{z^*} < 10\% \), then we stop.

Otherwise, we continue with the next iteration.

Monte Carlo Sampling

(k)th approximation

Empirical CDF

limit-state function

Updating Algorithm

Sampling Algorithm

(k+1)th approximation

Updating Algorithm

While \( \chi^{(k)}_{z^*} < 10\% \)

Set \( k = k+1 \)
CAARC standard tall building model

Table 1: Design of column members and beam members

<table>
<thead>
<tr>
<th>Floor zone</th>
<th>Column members</th>
<th>Beam members</th>
</tr>
</thead>
<tbody>
<tr>
<td>1~9F</td>
<td>W14X550</td>
<td>W30X357</td>
</tr>
<tr>
<td>10~18F</td>
<td>W14X500</td>
<td>W30X326</td>
</tr>
<tr>
<td>19~27F</td>
<td>W14X370</td>
<td>W30X292</td>
</tr>
<tr>
<td>28~36F</td>
<td>W14X257</td>
<td>W30X261</td>
</tr>
<tr>
<td>37~45F</td>
<td>W14X159</td>
<td>W30X221</td>
</tr>
</tbody>
</table>

- 180m × 30m × 45m
- 45-story, 10-bay by 15-bay
- Story height 4 m
- Bay width 3 m
- Floor
  - mass $6.75 \times 10^5$ kg
  - rotational moment of inertia $1.645 \times 10^8$ kg.m$^2$
Example

Wind excitation:

Table 2: Acting heights and acting areas of 6 excitation forces in the discretization scheme

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Acting height (m)</th>
<th>Acting area (m²)</th>
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<tr>
<td>U₁(t)</td>
<td>24</td>
<td>45×45</td>
</tr>
<tr>
<td>U₂(t)</td>
<td>68</td>
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</tr>
<tr>
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<td>33.75×45</td>
</tr>
<tr>
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<td>22.5×45</td>
</tr>
<tr>
<td>U₅(t)</td>
<td>156</td>
<td>22.5×45</td>
</tr>
<tr>
<td>U₆(t)</td>
<td>176</td>
<td>11.25×45</td>
</tr>
</tbody>
</table>

Wind excitation forces:

\[ U_j(t) = \frac{1}{2} \rho A_j (\bar{V}_j + v_j(t))^2 \]

Air density \( \rho = 1.2 \) kg/m³

Acting area \( A_j \)

Mean wind speed \( \bar{V}_j = 41 \) (√h / 180) 0.25

Fluctuating components \( v_j(t) \) ← SR method

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Example

Wind excitation: The along-wind excitation in the $Y$-direction of the building
Example

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Example

Wind excitation: The along-wind excitation in the $Y$-direction of the building

Wind excitation forces:

$$U_j(t) = \frac{1}{2} \rho A_j (\bar{V}_j + v_j(t))^2$$

- Air density $\rho = 1.2 \text{ kg/m}^3$
- Acting area $A_j$
- Mean wind speed $\bar{V}_j = 41 \left( \frac{h_j}{180} \right)^{0.25}$
- Fluctuating components $v_j(t) \leftarrow \text{SR method}$
Example

Structure response

\[ Y(t) = \sum_{j=1}^{\infty} \int_0^t q_j(t,\tau) U_j(\tau) d\tau \]

\( q_j(t,\tau) \) is the response function for \( Y(t) \).
Example

Structure response

Displacement response $Y(t)$ at the top floor of the building is of interest

$$Y(t) = \sum_{j=1}^{6} \int_{0}^{t} q_j(t, \tau) U_j(\tau) d\tau$$

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Example

Failure domain
Example

Failure domain

\[ \Omega : |Y(t)| \text{ exceeds a specified threshold } z^* \text{ within one hour.} \]
Example

Failure domain
\( \Omega: |Y(t)| \) exceeds a specified threshold \( z^* \) within one hour.

- Discrete time formulation
  - Duration time \( T = 3600 \) s
  - Sampling time interval \( \Delta t = 0.01 \) s
  - Number of time instants \( N_t = T/\Delta t = 3.6 \times 10^5 \)
Example

Failure domain

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\[
\Omega = \bigcup_{i=1}^{N_t} \left\{ x \in \mathbb{R}^N : |Y(i)| > z^* \right\}
\]

\( N = 2 \times N_u \times N_\omega = 8640 \)

\( g(x) = \max\{|Y(i)|, i = 1, \ldots, N_t\} \)
Example

Simulation results

1. $z^*_{1} = 1.25 \, \text{m} \Rightarrow p_{\Omega_{1}} = 3.3 \times 10^{-2}$, $\delta_{1} = 5.4\%$, $n_{MC} = 10^{4}$. 

2. $z^*_{2} = 1.35 \, \text{m} \Rightarrow p_{\Omega_{2}} = 6.8 \times 10^{-3}$, $\delta_{2} = 12.1\%$, $n_{MC} = 10^{4}$. 

3. $z^*_{3} = 1.45 \, \text{m} \Rightarrow p_{\Omega_{3}} = 1.6 \times 10^{-3}$, $\delta_{3} = 25.8\%$, $n_{MC} = 10^{4}$. 

Horseracing Simulation algorithm is applied with $n = 500$ initial samples.
Example

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Horseracing Simulation algorithm is applied with \( n = 500 \) initial samples

1. HRS\( \approx \)SS\( \approx \)MC
Example

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Horseracing Simulation algorithm is applied with \( n = 500 \) initial samples

1. \( \text{HRS} \approx \text{SS} \approx \text{MC} \)
2. Reductions in CV
   \[ \frac{\delta_2^{SS} - \delta_2^{HRS}}{\delta_2^{SS}} = 9.3\% \]
3. Reductions in CV
   \[ \frac{\delta_3^{SS} - \delta_3^{HRS}}{\delta_3^{SS}} = 14.4\% \]
Conclusion

- A new advanced stochastic simulation algorithm, called Horseracing Simulation (HRS), is proposed for solving high-dimensional reliability problems.
- The key idea behind HRS is to approximate the CDF of the random variable of interest by empirical CDFs constructed from specially designed samples.
- The accuracy and efficiency of the new method is demonstrated with real-life example.
Thank You for attention!