Bayesian Subset Simulation

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Joint work with J.L. Beck (Caltech)

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Outline

1. Reliability Problem
2. Original Subset Simulation method
3. Bayesian Subset Simulation
4. Example
5. Summary

Reliability Problem

Reliability Problem: To estimate the probability of failure p_F

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Notation:

- $\theta \in \mathbb{R}^d$ represents the uncertain excitation of a system
  - θ is a random vector with joint PDF $\pi(\theta)$
- $F \subset \mathbb{R}^d$ is a failure domain (unacceptable system performance)

$$F = \{\theta : g(\theta) \geq b^*\}$$

- $g(\theta)$ is a performance function (loss function)
- b^* is a critical threshold for performance
- $I_F(\theta) = 1$ if $\theta \in F$; and $I_F(\theta) = 0$ if $\theta \notin F$

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We need advanced simulation methods

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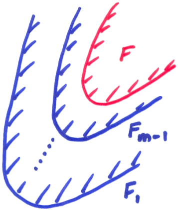

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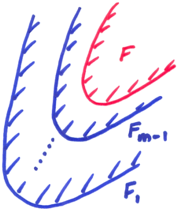

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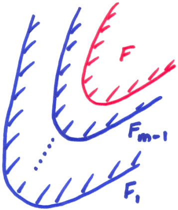
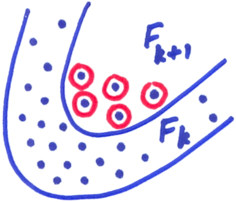
$$P(F_{k+1}|F_k) \approx \frac{1}{N}\sum_{i=1}^{N} I_{F_{k+1}}(\theta_k^{(i)})$$

$$\theta_k^{(i)} \sim \pi(\theta|F_k) = \frac{\pi(\theta) I_{F_k}(\theta)}{P(F_k)}$$





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3. Obtain the posterior PDF $f(p_F|\cup_{k=0}^{m-1}\mathcal{D}_k)$ of $p_F = \prod_{k=1}^{m} p_k$ from $f(p_1|\mathcal{D}_0), \ldots, f(p_m|\mathcal{D}_{m-1})$.

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     $\Rightarrow$ Bayes' Theorem (1763):

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   - In fact, $\theta_{k-1}^{(1)}, \ldots, \theta_{k-1}^{(N)}$ are MCMC samples (for $k \ge 2$)
     $\Rightarrow \theta_{k-1}^{(1)}, \ldots, \theta_{k-1}^{(N)} \sim \pi(\cdot|F_{k-1})$, however, they are not independent
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Posterior PDF for p_F

Last step: To find the PDF of $p_F = \prod_{k=1}^{m} p_k$, given the PDFs of all factors

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Theorem (Da-Yin Fan, 1991)

Let $X_1, \ldots, X_m$ be beta variables, $X_k \sim \mathcal{B}e(a_k, b_k)$, and $Y = X_1 X_2 \ldots X_m$. Then Y is approximately distributed as $\widetilde{Y} \sim \mathcal{B}e(a, b)$, where

$$a = \mu_1 \frac{\mu_1 - \mu_2}{\mu_2 - \mu_1^2}, \qquad b = (1 - \mu_1) \frac{\mu_1 - \mu_2}{\mu_2 - \mu_1^2},$$

$$\mu_1 = \prod_{k=1}^m \frac{a_k}{a_k + b_k}, \qquad \mu_2 = \prod_{k=1}^m \frac{a_k(a_k + 1)}{(a_k + b_k)(a_k + b_k + 1)}.$$

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Nice property of this approximation: $\mathbb{E}[\widetilde{Y}] = \mathbb{E}[Y]$, $\mathbb{E}[\widetilde{Y}^2] = \mathbb{E}[Y^2]$

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- Why is Bayesian Subset Simulation useful?
  - CV of $f(p_F)$ can be considered as a measure of uncertainty in the value of p_F
  - The PDF $f(p_F)$ can be fully used for life-cost analyses, decision making, etc.

$$\mathbb{E}[\mathrm{Loss}(p_F)] = \int \mathrm{Loss}(p_F)f(p_F)dp_F$$

Elasto-Plastic Structure Subjected to Ground Motion

S.K. Au (Computers & Structures, 2005):

- 2D moment-resisting steel frame
- Synthetic ground motion $a = a(Z)$
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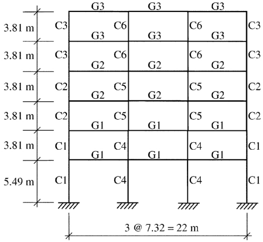
  - $\xrightarrow{Z}$ Filter $\xrightarrow{a(Z)}$
  - $d = 1001$
- Failure domain:

$$F = \{Z \in \mathbb{R}^d : \delta_{\max}(Z) > b\}$$

$$\delta_{\max} = \max_{i=1,\dots,6} \delta_i$$

δ_i is the maximum absolute interstory drift ratio of the i^{th} story within the duration of study, 30 s

$$b = 0.5\% \Rightarrow p_F \approx 8.9 \times 10^{-3}$$



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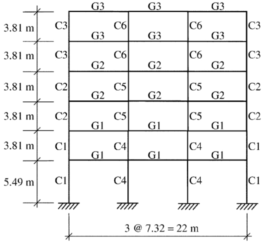

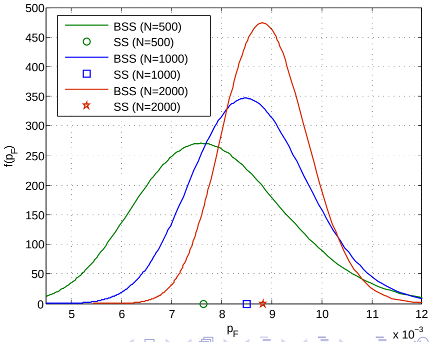

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$$\lim_{N\to\infty} \mathbb{E}_f[p_F] = \lim_{N\to\infty} \widehat{p}_F = p_F$$

- CV of $f(p_F)$ can be considered as a measure of uncertainty in the value of p_F

Summary

Bayesian Subset Simulation

- BSS is a new stochastic simulation method for solving reliability problems.
  - BSS is a Bayesian analog of the Subset Simulation (Au and Beck, 2001)
- Instead of a point estimate $\widehat{p}_F$, BSS produces an approximation of the posterior PDF $f(p_F)$ of the failure probability.
- Relationship between BSS and SS:

$$\lim_{N\to\infty} \mathbb{E}_f[p_F] = \lim_{N\to\infty} \widehat{p}_F = p_F$$

- CV of $f(p_F)$ can be considered as a measure of uncertainty in the value of p_F
- The PDF $f(p_F)$ can be fully used for life-cost analyses and decision making.

$$\mathbb{E}[\mathrm{Loss}(p_F)] = \int \mathrm{Loss}(p_F) f(p_F) dp_F$$

Thank you for attention!