Bayesian Subset Simulation for failure probability estimation

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Outline

1. Dynamic Reliability Problem
2. Original Subset Simulation method
3. Bayesian Subset Simulation
   - General strategy of "Bayesianization"
   - Prior and Posterior for $p_k = P(F_k|F_{k-1})$
   - Products of stochastic variables
   - Posterior PDF of p_F
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4. Example
5. Summary

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We need advanced simulation methods

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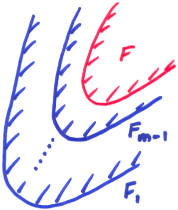

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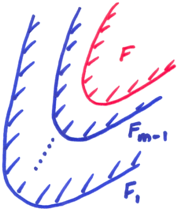

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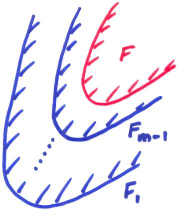
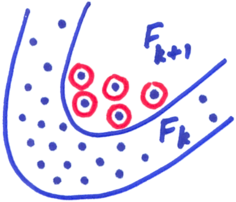
$$P(F_{k+1}|F_k) \approx \frac{1}{N} \sum_{i=1}^{N} I_{F_{k+1}}(\theta_k^{(i)})$$

$$\theta_k^{(i)} \sim \pi(\theta|F_k) = \frac{\pi(\theta) I_{F_k}(\theta)}{P(F_k)}$$





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$$p_k \approx \hat{p}_k = \frac{1}{N}\sum_{i=1}^{N} I_{F_k}(\theta_{k-1}^{(i)}) = \frac{n_k}{N}, \qquad p_F \approx \hat{p}_F^{SS} = \prod_{k=1}^{m} \frac{n_k}{N}$$

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2. Find the posterior PDFs $p(p_k|\mathcal{D}_{k-1})$ via Bayes' theorem, using new data $\mathcal{D}_{k-1} = \{\theta_{k-1}^{(1)}, \ldots, \theta_{k-1}^{(N)} \sim \pi(\cdot|F_{k-1})\}$

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3. Obtain the posterior PDF $p(p_F|\cup_{k=0}^{m-1}\mathcal{D}_k)$ of $p_F = \prod_{k=1}^{m} p_k$ from $p(p_1|\mathcal{D}_0), \ldots, p(p_m|\mathcal{D}_{m-1})$.

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   - In fact, $\theta_{k-1}^{(1)}, \dots, \theta_{k-1}^{(N)}$ are MCMC samples (for $k \geq 2$)
     $\Rightarrow \theta_{k-1}^{(1)}, \dots, \theta_{k-1}^{(N)} \sim \pi(\cdot|F_{k-1})$, however, they are not independent

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Products of stochastic variables

Last step: To find the PDF of the product of stochastic variables $p_F = \prod_{k=1}^{m} p_k$, given the distribution of all factors p_k

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Theorem (Rohatgi's formula)

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Theorem (Tang and Gupta, 1984)

Let $X_1, \ldots, X_m$ be beta variables, $X_k \sim Beta(a_k, b_k)$, and $Y = X_1 X_2 \ldots X_m$. Then the density function of Y can be written as follows:

$$f_Y(y) = \left(\prod_{k=1}^{m} \frac{\Gamma(a_k + b_k)}{\Gamma(a_k)} \right) y^{a_m - 1} (1-y)^{\sum_{k=1}^{m} b_k - 1} \cdot \sum_{r=0}^{\infty} \sigma_r^{(m)} (1-y)^r, \quad 0 < y < 1.$$

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Let $p^{BSS}(p_F| \cup_{k=0}^{m-1} \mathcal{D}_k)$ denote the posterior of p_F obtained from the theorem.

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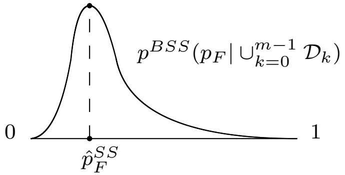
$$\hat{p}_F^{SS} = p_{MAP}^{BSS}$$

$$p_{MAP}^{BSS} = \arg \max_{p_F \in [0,1]} p^{BSS}(p_F | \cup_{k=0}^{m-1} \mathcal{D}_k)$$



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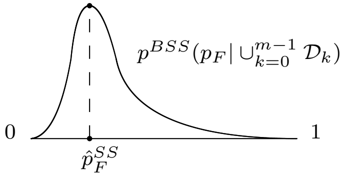

Drawback of $p^{BSS}(p_F| \cup_{k=0}^{m-1} \mathcal{D}_k)$: Its expression contains an infinite sum.

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$$a = \mu_1 \frac{\mu_1 - \mu_2}{\mu_2 - \mu_1^2}, \qquad b = (1 - \mu_1)\frac{\mu_1 - \mu_2}{\mu_2 - \mu_1^2},$$

$$\mu_1 = \prod_{k=1}^{m} \frac{a_k}{a_k + b_k}, \qquad \mu_2 = \prod_{k=1}^{m} \frac{a_k(a_k + 1)}{(a_k + b_k)(a_k + b_k + 1)}.$$

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Nice property of this approximation:

$$\mathbb{E}[\tilde{Y}] = \mathbb{E}[Y]$$

$$\mathbb{E}[\tilde{Y}^2] = \mathbb{E}[Y^2]$$

Approximation $\tilde{p}^{BSS}$ of the posterior PDF of p_F

$$p^{BSS}(p_F|\cup_{k=0}^{m-1}\mathcal{D}_k) \approx \tilde{p}^{BSS}(p_F|\cup_{k=0}^{m-1}\mathcal{D}_k) = Beta(p_F|a,b)$$

$$a = \frac{\prod_{k=1}^{m}\frac{n_k+1}{N+2}\left(1-\prod_{k=1}^{m}\frac{n_k+2}{N+3}\right)}{\prod_{k=1}^{m}\frac{n_k+2}{N+3}-\prod_{k=1}^{m}\frac{n_k+1}{N+2}}$$

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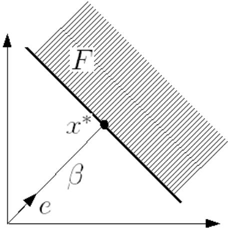
What is the relationship between $\tilde{p}^{BSS}$ and $\hat{p}_F^{SS}$?

$\mathbb{E}_{\tilde{p}^{BSS}}[p_F] \rightarrow \hat{p}_F^{SS}$, as $N \rightarrow \infty$

$\mathbb{E}_{\tilde{p}^{BSS}}[p_F] \approx \hat{p}_F^{SS}$, when N is large

Example

Linear problem



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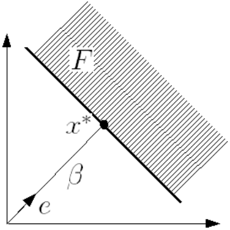

Geometry

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- $\pi(x) = \mathcal{N}(0, I_N)$
- $p_F = 10^{-3}$, $\beta = 3.09$

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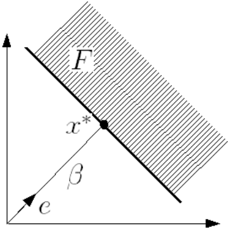

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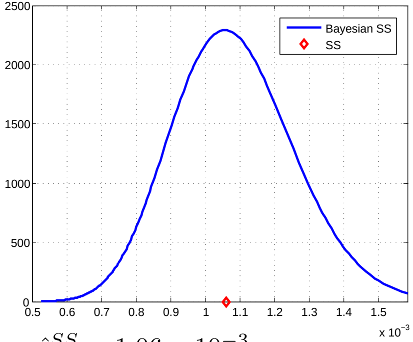
BSS vs SS



- $\hat{p}_F^{SS} = 1.06 \times 10^{-3}$
- $\tilde{p}_{MAP}^{BSS} = 1.05 \times 10^{-3}$
- $\mathbb{E}_{\tilde{p}^{BSS}}[p_F] = 1.08 \times 10^{-3}$
- $\delta = \frac{\sqrt{\mathbb{V}ar_{\tilde{p}^{BSS}}[p_F]}}{\mathbb{E}_{\tilde{p}^{BSS}}[p_F]} = 0.16$

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$$\mathbb{E}_{\tilde{p}^{BSS}}[p_F] \rightarrow \hat{p}_F^{SS}, \text{ as } N \rightarrow \infty$$

- The PDF $\tilde{p}^{BSS}$ can be fully used for life-cost analyses, decision making, etc.

$$\mathbb{E}[\text{Loss}(p_F)] = \int \text{Loss}(p_F)\tilde{p}^{BSS}(p_F)dp_F$$

Thank you for attention!