Computing concordance invariants from involutive Heegaard Floer homology

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The SURF project we will carry out is in the area of low-dimensional topology. The basic tools in this field are algebraic/differential topology and combinatorics. In our project, we also expect to use computers to do some computations.

A *knot* is an embedded cirlce in S^3. Two knots $K_0, K_1 \subset S^3$ are (smoothly) *concordant* if there exists a smoothly embedded annulus $A \subset S^3 \times [0,1]$, such that $A \cap (S^3 \times \{i\}) = K_i \times \{i\}$, $i = 0, 1$. A knot is (smoothly) *slice* if it is concordant to the unknot.

It is a very subtle and difficult problem to study the concordance relation of knots. Even determining whether a knot is slice is highly nontrivial. One example is the $(1,2)$–cable of the figure-8 knot. It is unknown whether this knot is slice or not. None of the old concordance invariants yield any information on the sliceness of this knot.

The first goal of this project is to compute two new concordance invariants $\overline{V}_0, \underline{V}_0$ from Hendricks and Manolescu's Involutive Heegaard Floer homology [4]. These two invariants are algorithmically computable. In fact, using [5], one can always compute these invariants. However, the complexity of the algorithm is an issue. There do exist many other methods to compute the invariants, and we will investigate them.

If we finish the case of the $(1,2)$–cable of the figure-8 knot, we will move on to other similar knots.

We will recruit 1 to 2 students to work on this project. Ma 5 and Ma 109 are required. The students are also expected to have some basic knowledge of Ma 151. In order to be considered for this project, the applicant should send his/her CV and transcript to Yi Ni by February 8, and schedule a 30-minute presentation of the paper [5] to Yi Ni. The presentation should be finished by February 15.

The students should learn some basic knot theory and homology theory by the beginning of the SURF period. (Suggested readings: [1] and [2, Chapter 2].)

During the SURF period, we will have two group meetings every week. Each meeting will be one or two hours. In the first three weeks, we will read some papers about knot Floer homology, including [6, 3, 4]. Every student is expected to give 2 to 4 one-hour presentations on the papers. In the next four to six weeks, we will try to compute the concordance invariants for the cable knot we mentioned. Time permitted, we will also study other knots.

References

[1] **C. Adams**, *The knot book. An elementary introduction to the mathematical theory of knots.* W. H. Freeman and Company, New York, 1994. xiv+306 pp.

[2] **A. Hatcher**, *Algebraic topology*, Cambridge University Press, Cambridge, 2002. xii+544 pp.

[3] **M. Hedden**, *On knot Floer homology and cabling*, Algebr. Geom. Topol. 5 (2005), 1197–1222.

[4] **K. Hendricks, C. Manolescu**, *Involutive Heegaard Floer homology*, Duke Math. J. 166 (2017), no. 7, 1211–1299.

[5] **C. Manolescu, P. Ozsváth, S. Sarkar**, *A combinatorial description of knot Floer homology*, Ann. of Math. (2) 169 (2009), no. 2, 633–660.

[6] **P. Ozsváth, Z. Szabó**, *Holomorphic disks and knot invariants*, Adv. Math. 186 (2004), no. 1, 58–116.