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HEEGAARD FLOER HOMOLOGY AND FIBRED 3-MANIFOLDS

By Yi Ni

Dedicated to the memory of Xiao-Song Lin.

Abstract. Given a closed 3-manifold Y, we show that the Heegaard Floer homology determines whether Y fibres over the circle with a fibre of negative Euler characteristic. This is an analogue of an earlier result about knots proved by Ghiggini and the author.

1. Introduction. Heegaard Floer homology was introduced by Ozsváth and Szabó in [11]. This theory contains a package of invariants for closed 3-manifolds. A filtered version of these invariants, called knot Floer homology, was defined by Ozsváth–Szabó [13] and Rasmussen [16] for null-homologous knots. This theory turns out to be very powerful. For example, it detects the Thurston norm [15], and a result due to Ghiggini [4] and the author [9] states that knot Floer homology detects fibred knots.

In fact, given a compact manifold with boundary, the information from knot Floer homology tells you whether this manifold is fibred. (See the proof of [9, Corollary 1.2].) Now it is natural to ask whether a similar result can be proved for closed manifolds. In the current paper, we will answer this question affirmatively. Our main theorem is:

Theorem 1.1. *Suppose Y is a closed irreducible 3-manifold, $F \subset Y$ is a closed connected surface of genus $g \geq 2$. Let $HF^+(Y, [F], g-1)$ denote the group*

$$\bigoplus_{\mathfrak{s} \in \mathrm{Spin}^c(Y),\ \langle c_1(\mathfrak{s}), [F] \rangle = 2g-2} HF^+(Y, \mathfrak{s}).$$

If $HF^+(Y, [F], g-1) \cong \mathbb{Z}$, then Y fibres over the circle with F as a fibre.

The converse of this theorem is already known, see [14, Theorem 5.2].

Remark 1.2. When $g = 1$, there is no chance for $HF^+(Y, [F], 0)$ to be $\mathbb{Z}$; some arguments in this paper also break down. Nevertheless, it is reasonable

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to expect that Heegaard Floer homology still detects fibrations in the case of $g = 1$. As suggested by Ozsváth and Szabó, in this case one may use some variants of Heegaard Floer homology, for instance, Heegaard Floer homology with twisted coefficients [12, Section 8] or with coefficients in the Novikov ring [11, 11.0.1], [5].

This paper is closely related to the previous works of Ghiggini [4] and of the author [9]. We briefly recall the main ingredients in the proof of the case of knots from those two papers. There are three main ingredients:

(I) The construction of two different taut foliations under certain conditions. This part is contained in [9, Section 6] and the proof uses an argument due to Gabai [3]. This part was also obtained by Ian Agol by a different method, and the genus 1 case was proved in [4].

(II) The existence of a taut foliation implies the nontriviality of the corresponding Ozsváth–Szabó contact invariant. This part was almost proved by Ozsváth and Szabó [15]. But there are some technical restrictions in the cases established by Ozsváth and Szabó. For example, one needs the condition $b_1(Y) = 1$ or the use of twisted coefficients. The case that is used in [4], [9] was proved by Ghiggini in [4]. Basically, he proved a result in contact topology, which, combined with some results of Ozsváth and Szabó, implies the desired nontriviality theorem.

(III) Two decomposition formulas for knot Floer homology, one in the case of horizontal decomposition [9, Theorem 4.1], the other in the case of decomposition along a separating product annulus [9, Theorem 5.1]. The second formula is essential and more technical. The proof of this part uses the techniques introduced in [8].

Now if (I) can be done, then (II) implies that the two distinct taut foliations give rise to two linearly independent contact invariants in the topmost term of the Heegaard Floer homology, so the Floer homology is not monic. If (I) cannot be done, then the topology of the knot complement is very restricted. One can then use (III) to reduce the general case to the known case.

As we will find in this paper, the above ingredients (I) and (II) can be applied to the case of closed manifolds without essential changes. However, we are not able to prove an analogue of [9, Theorem 5.1] for closed manifolds. Instead, we will construct a knot K in a new manifold Z, and show that the pair (Z, K) has monic knot Floer homology by using twisted coefficients and a simple argument in homological algebra. Then we can apply [9, Theorem 1.1] to get our conclusion.

Remark 1.3. Juhász [6] proved a very general decomposition formula for the Floer homology of balanced sutured manifolds, based on the techniques introduced by Sarkar and Wang [17]. In the case of knots, the above ingredients (II) and (III) can be deduced from this formula. This approach avoids the use of contact and symplectic topology, but it is not clear to the author how to use it to study closed manifolds.

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2. Twisted Heegaard Floer homology. For technical reasons, at some point in the proof of our main theorem we will use twisted Heegaard Floer homology with coefficients in a Novikov ring. In this section, we will collect some basic materials on this version of twisted Heegaard Floer homology. More details about twisted Heegaard Floer homology can be found in [12], [5], [1].

2.1. Twisted chain complexes. Let Y be a closed, oriented 3-manifold. $(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta}, z)$ is a Heegaard diagram for Y. We always assume the diagram satisfies a certain admissibility condition so that the Heegaard Floer invariants we are considering are well-defined (see [11] for more details).

Let

$$\mathcal{L} = \mathbb{Q}[T^{-1}, T]] = \left\{ \sum_{i=n}^{+\infty} a_i T^i \,\middle|\, a_i \in \mathbb{Q},\ n \in \mathbb{Z} \right\}$$

be a Novikov ring, which is actually a field.

Let $\omega \subset Y$ be an immersed, possibly disconnected, closed, oriented curve. One can homotope ω to be a curve $\omega' \subset \Sigma$, such that ω' is in general position with the α- and β-curves, namely, ω' is transverse to these curves, and ω' does not contain any intersection point of α- and β-curves.

Let $\underline{CF}^{\infty}(Y, \omega; \mathcal{L})$ be the $\mathcal{L}$-module freely generated by $[\mathbf{x}, i]$, where $\mathbf{x} \in \mathbb{T}_{\alpha} \cap \mathbb{T}_{\beta}$, $i \in \mathbb{Z}$. If ϕ is a topological Whitney disk connecting $\mathbf{x}$ to $\mathbf{y}$, let $\partial_{\alpha}\phi = (\partial\phi) \cap \mathbb{T}_{\alpha}$. We can also regard $\partial_{\alpha}\phi$ as a multi-arc that lies on Σ and connects $\mathbf{x}$ to $\mathbf{y}$. We define

$$A(\phi) = (\partial_{\alpha}\phi) \cdot \omega'.$$

Geometrically, if two Whitney disks ϕ_1, ϕ_2 differ by a periodic domain $\mathcal{P}$, then

$$A(\phi_1) - A(\phi_2) = H(\mathcal{P}) \cdot \omega,$$

where $H(\mathcal{P}) \in H_2(Y)$ is the homology class represented by $\mathcal{P}$.

Let

$$\underline{\partial}\colon\ \underline{CF}^{\infty}(Y,\omega;\mathcal{L}) \to \underline{CF}^{\infty}(Y,\omega;\mathcal{L})$$

be the boundary map defined by

$$\underline{\partial}\,[\mathbf{x},i] = \sum_{\mathbf{y}} \sum_{\substack{\phi\in\pi_2(\mathbf{x},\mathbf{y})\\ \mu(\phi)=1}} \#\left(\mathcal{M}(\phi)/\mathbb{R}\right) T^{A(\phi)}[\mathbf{y}, i-n_z(\phi)].$$

The chain homotopy type of the chain complex

$$\left(\underline{CF}^{\infty}(Y,\omega;\mathcal{L}),\underline{\partial}\right)$$

only depends on the homology class $[\omega]\in H_1(Y)$. When ω is null-homologous in Y, the coefficients are not "twisted" at all, namely,

$$\underline{CF}^{\infty}(Y,\omega;\mathcal{L}) \cong CF^{\infty}(Y;\mathbb{Q})\otimes_{\mathbb{Q}}\mathcal{L}.$$

The standard construction in Heegaard Floer homology [11] allows us to define the chain complexes $\underline{\widehat{CF}}(Y,\omega;\mathcal{L})$ and $\underline{CF}^{\pm}(Y,\omega;\mathcal{L})$. When K is a null-homologous knot in Y and $\omega\subset Y-K$, we can define the twisted knot Floer complex $\underline{\widehat{CFK}}(Y,K,\omega;\mathcal{L})$. The homologies of the chain complexes are called twisted Heegaard Floer homologies.

2.2. Twisted chain maps. Let $(\Sigma,\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},z)$ be a Heegaard triple-diagram. Let $\omega'\subset\Sigma$ be a closed immersed curve which is in general position with the α-, β- and γ-curves.

The pants construction in [11, Subsection 8.1] gives rise to a four-manifold $X_{\alpha,\beta,\gamma}$ with

$$\partial X_{\alpha,\beta,\gamma} = -Y_{\alpha,\beta} - Y_{\beta,\gamma} + Y_{\alpha,\gamma}\,.$$

By this construction $X_{\alpha,\beta,\gamma}$ contains a region $\Sigma\times\triangle$, where $\triangle$ is a two-simplex with edges $e_\alpha, e_\beta, e_\gamma$. Let $\omega'\times[0,1]=\omega'\times e_\alpha\subset X_{\alpha,\beta,\gamma}$ be the natural properly immersed annulus such that

$$\omega'\times\{0\}\subset Y_{\alpha,\beta}\,,\quad \omega'\times\{1\}\subset Y_{\alpha,\gamma}\,.$$

Suppose $\mathbf{x}\in\mathbb{T}_\alpha\cap\mathbb{T}_\beta, \mathbf{y}\in\mathbb{T}_\beta\cap\mathbb{T}_\gamma, \mathbf{w}\in\mathbb{T}_\alpha\cap\mathbb{T}_\gamma$, ψ is a topological Whitney triangle connecting them. Let $\partial_\alpha\psi=\partial\psi\cap\mathbb{T}_\alpha$ be the arc connecting $\mathbf{x}$ to $\mathbf{w}$. We can regard $\partial_\alpha\psi$ as a multi-arc on Σ. Define

$$A_3(\psi) = (\partial_\alpha\psi)\cdot\omega'.$$

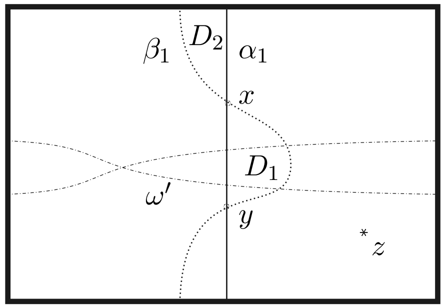

Figure 1. A Heegaard diagram for $S^2 \times S^1$. The Heegaard surface is the torus obtained by gluing the opposite sides of the rectangle.

Let the chain map

$$\underline{f^{\infty}_{\alpha,\beta,\gamma,\,\omega'\times I}}\colon\ \underline{CF}^{\infty}(Y_{\alpha,\beta},\omega'\times\{0\};\mathcal{L})\otimes_{\mathbb{Q}} CF^{\infty}(Y_{\beta,\gamma};\mathbb{Q})\to \underline{CF}^{\infty}(Y_{\alpha,\gamma},\omega'\times\{1\};\mathcal{L})$$

be defined by the formula:

$$\underline{f^{\infty}_{\alpha,\beta,\gamma,\,\omega'\times I}}([\mathbf{x},i]\otimes[\mathbf{y},j]) = \sum_{\mathbf{w}} \sum_{\substack{\psi\in\pi_2(\mathbf{x},\mathbf{y},\mathbf{w})\\ \mu(\psi)=0}} \#\mathcal{M}(\psi)T^{A_3(\psi)}[\mathbf{w}, i+j-n_z(\psi)].$$

The standard constructions [11], [12] allow us to define chain maps introduced by cobordisms. We also have the surgery exact triangles. For example, suppose $K\subset Y$ is a knot with frame λ, $\omega\subset Y-K$ is a closed curve, then ω also lies in the manifolds Y_λ and $Y_{\lambda+\mu}$ obtained by surgeries. The 2-handle addition cobordism W from Y to Y_λ naturally contains a properly immersed annulus $\omega\times I$. We can define a chain map induced by W:

$$\underline{f^{\infty}_{W,\,\omega\times I}}\colon\ \underline{CF}^{\infty}(Y,\omega;\mathcal{L})\to\underline{CF}^{\infty}(Y_\lambda,\omega;\mathcal{L}).$$

Similarly, there are two other chain maps induced by the cobordisms $Y_\lambda\to Y_{\lambda+\mu}$ and $Y_{\lambda+\mu}\to Y$. We then have the long exact sequence [1]:

$$(1)\qquad \cdots\to\underline{HF}^{+}(Y,\omega;\mathcal{L})\to\underline{HF}^{+}(Y_\lambda,\omega;\mathcal{L})\to\underline{HF}^{+}(Y_{\lambda+\mu},\omega;\mathcal{L})\to\cdots.$$

2.3. Some properties of twisted Heegaard Floer homology. Many properties of the untwisted Heegaard Floer homology have analogues in the twisted case. For example, the connected sum formula for $\underline{\widehat{HF}}$ is the following:

$$\underline{\widehat{HF}}(Y_1,\omega_1;\mathcal{L})\otimes\underline{\widehat{HF}}(Y_2,\omega_2;\mathcal{L})\cong\underline{\widehat{HF}}(Y_1\#Y_2,\omega_1\cup\omega_2;\mathcal{L}).$$

LEMMA 2.1. *Suppose Y contains a nonseparating two-sphere S, $\omega \in Y$ is a closed curve such that $\omega \cdot S \neq 0$. We then have*

$$\underline{\widehat{HF}}(Y, \omega; \mathcal{L}) = 0, \quad \underline{HF}^+(Y, \omega; \mathcal{L}) = 0.$$

Proof. As in Figure 1, $\underline{\widehat{CF}}(S^2 \times S^1, \omega; \mathcal{L})$ has two generators x, y. There are two bigons D_1, D_2 connecting x to y. If $\omega \cdot (S^2 \times \text{point}) = d \neq 0$, we can assume $A(D_1) = d, A(D_2) = 0$, then $\underline{\partial} x = \pm(T^d - 1)y$. This implies that $\underline{\widehat{HF}}(S^1 \times S^2, \omega; \mathcal{L}) = 0$ since $T^d - 1$ is invertible in $\mathcal{L}$.

If Y contains a nonseparating two-sphere S and $\omega \cdot S \neq 0$, then Y has a summand $S^2 \times S^1$ such that $\omega \cdot (S^2 \times \text{point}) \neq 0$. The connected sum formula then shows that $\underline{\widehat{HF}}(Y, \omega; \mathcal{L}) = 0$.

For $\underline{HF}^+$, it follows from the long exact sequence

$$\cdots \longrightarrow \underline{HF}^+ \xrightarrow{U} \underline{HF}^+ \longrightarrow \underline{\widehat{HF}} \longrightarrow \cdots$$

that U is an isomorphism. For any element $x \in \underline{HF}^+(Y, \omega; \mathcal{L})$, $U^n x = 0$ when n is sufficiently large, so $\underline{HF}^+(Y, \omega; \mathcal{L}) = 0$. □

The following theorem is a twisted version of [9, Theorem 1.1].

THEOREM 2.2. *Suppose K is a null-homologous knot in a closed, oriented, connected 3-manifold Y, $Y - K$ is irreducible, and F is a genus g Seifert surface of K. Let $\omega \subset Y - K$ be a closed curve. If $\underline{\widehat{HFK}}(Y, K, \omega, [F], g; \mathcal{L}) \cong \mathcal{L}$, then K is fibred, and F is a fibre of the fibration.*

Proof. We could prove this theorem by repeating the whole proof in [9], but we would rather choose to apply [9, Theorem 1.1] directly.

Let (M, γ) be the sutured manifold (see [9, Definition 2.1]) obtained by cutting $Y - K$ open along F. The proof of [9, Proposition 3.1] shows that M is a homology product. Hence we can glue $R_+(\gamma)$ to $R_-(\gamma)$ by a suitable homeomorphism to get a manifold with torus boundary, which is the exterior of a knot K' in a manifold Y' with $b_1(Y') = 0$. This cut-and-reglue process can be realized by Dehn surgeries on knots in F, so ω can be regarded as a curve in $Y' - K'$. As in [8, Proposition 3.5, the second proof], using a filtered version of the exact sequence (1) and the adjunction inequality, we can show that $\underline{\widehat{HFK}}(Y', K', \omega, [F], g; \mathcal{L}) \cong \mathcal{L}$.

Since $b_1(Y') = 0$, there is no real "twisting" at all, namely,

$$\underline{\widehat{CFK}}(Y', K', \omega; \mathcal{L}) \cong \widehat{CFK}(Y', K'; \mathbb{Q}) \otimes_{\mathbb{Q}} \mathcal{L}.$$

So we get $\widehat{HFK}(Y', K', [F], g; \mathbb{Q}) \cong \mathbb{Q}$. Now [9, Theorem 1.1] implies that K' is fibred with fibre F, hence so is K. □

3. A homological version of the main theorem. The goal of this section is to prove the following homological version of the main theorem.

PROPOSITION 3.1. *Suppose Y is a closed 3-manifold, $F \subset Y$ is a closed non-separating connected surface of genus $g \geq 2$. Let M be the 3-manifold obtained by cutting Y open along F. The two boundary components of ∂M are denoted by F_-, F_+. If $HF^+(Y, [F], g-1) \cong \mathbb{Z}$, then M is a homology product, namely, the two maps*

$$i_{\pm *}\colon H_*(F_\pm) \to H_*(M)$$

are isomorphisms.

We will make use of the fact that the Euler characteristic of HF^+ is Turaev's torsion function. A Heegaard diagram for Y will be constructed, and will be used to study the torsion function of Y. Then Proposition 3.1 can be proved using the same argument as [9, Proposition 3.1].

3.1. A Heegaard diagram for *Y*.

Construction 3.2. We will construct a Heegaard diagram for Y in a similar manner as in [9, Construction 2.10].

Step 0. A relative Morse function. Let M be the compact manifold obtained by cutting open Y along F; the two components of M are denoted by F_-, F_+. Let $\psi\colon F_+ \to F_-$ be the gluing map. Consider a self-indexed relative Morse function u on M. Namely, u satisfies:

(1) $u(M) = [0,3]$, $u^{-1}(0) = F_-$, $u^{-1}(3) = F_+$.

(2) u has no degenerate critical points, $u^{-1}\{\text{critical points of index } i\} = i$.

(3) u has no critical points on $F_\pm$.

Let $F_\# = u^{-1}(\frac{3}{2})$.

Suppose u has r index 1 critical points. Then the genus of $F_\#$ is $g + r$. The gradient $-\nabla u$ generates a flow ϕ_t on M. There are $2r$ points on F_+, which are connected to index 2 critical points by flowlines. We call these points "bad" points. Similarly, there are $2r$ bad points on F_-, which are connected to index 1 critical points by flowlines.

Step 1. Construct some curves. Choose two disjoint disks $D_+^a, D_+^b \subset F_+$. Flow the two disks by ϕ_t, their images on $F_\#$ and F_- are $D_\#^a, D_-^a, D_\#^b, D_-^b$. (We choose the disks generically, so that the flowlines starting from them do not terminate at critical points.) We can suppose the gluing map ψ is equal to the intersection of the flow ϕ_t with F_- when restricted to $D_+^a \cup D_+^b$. Let $A_\pm = F_\pm - \mathrm{int}(D_\pm^a \cup D_\pm^b)$, $A_\# = F_\# - \mathrm{int}(D_\#^a \cup D_\#^b)$.

On $F_\#$, there are r simple closed curves $\alpha_{2g+1}, \ldots, \alpha_{2g+r}$, which are connected to index 1 critical points by flowlines. And there are r simple closed curves $\beta_{2g+1}, \ldots, \beta_{2g+r}$, which are connected to index 2 critical points by flowlines.

Choose $2g$ disjoint arcs $\xi_1^-, \ldots, \xi_{2g}^- \subset A_-$, such that their endpoints lie on ∂D_-^b, and they are linearly independent in $H_1(A_-, \partial A_-)$. We also suppose they are disjoint from the bad points. Let $\xi_i^+ = \psi^{-1}(\xi_i^-)$. We also flow back $\xi_1^-, \dots, \xi_{2g}^-$ by ϕ_{-t} to $F_\#$, the images are denoted by $\xi_1^\#, \dots, \xi_{2g}^\#$.

Choose $2g$ disjoint arcs $\eta_1^+, \dots, \eta_{2g}^+ \subset A_+$, such that their endpoints lie on ∂D_+^a, and they are linearly independent in $H_1(A_+, \partial A_+)$. We also suppose they are disjoint from the bad points. Flow them by ϕ_t to $F_\#$, the images are denoted by $\eta_1^\#, \dots, \eta_{2g}^\#$.

Step 2. Construct a diagram. Suppose $[c_1, c_2]$ is a subinterval of $[0, 3]$, let $(\partial D_+^a) \times [c_1, c_2]$ be the annulus which is the image of ∂D_+^a inside $u^{-1}([c_1, c_2])$ under the flow ϕ_t. Similarly, define $(\partial D_+^b) \times [c_1, c_2]$. Let

$$\Sigma = A_+ \cup A_\# \cup \left((\partial D_+^a) \times \left[\frac{3}{2}, 3\right]\right) \cup \left((\partial D_+^b) \times \left[0, \frac{3}{2}\right]\right).$$

Let

$$\alpha_i = \xi_i^+ \cup \xi_i^\# \cup \{2 \text{ arcs}\},$$

where the 2 arcs are vertical arcs connecting ξ_i^+ to $\xi_i^\#$ on a corresponding annulus, $i = 1, \dots, 2g$. Similarly, let

$$\beta_i = \eta_i^+ \cup \eta_i^\# \cup \{2 \text{ arcs}\}.$$

Let $\alpha_0 = \partial D_\#^a$, $\beta_0 = \partial D_\#^b$.

Let

$$\begin{aligned} \boldsymbol{\alpha} &= \{\alpha_1, \dots, \alpha_{2g}\} \cup \{\alpha_{2g+1}, \dots, \alpha_{2g+r}\} \cup \{\alpha_0\}, \\ \boldsymbol{\beta} &= \{\beta_1, \dots, \beta_{2g}\} \cup \{\beta_{2g+1}, \dots, \beta_{2g+r}\} \cup \{\beta_0\}. \end{aligned}$$

Step 3. Check that $(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta})$ is a Heegaard diagram for Y. This step is quite routine, we leave the reader to check the following

(A) Σ separates Y into two genus $(2g + 1 + r)$ handlebodies U_1, U_2. Every curve in $\boldsymbol{\alpha}$ bounds a disk in U_1, every curve in $\boldsymbol{\beta}$ bounds a disk in U_2.

(B) $\Sigma - \boldsymbol{\alpha}$ is connected, $\Sigma - \boldsymbol{\beta}$ is connected.

Then

$$(\Sigma, \boldsymbol{\alpha}, \boldsymbol{\beta})$$

is a Heegaard diagram for Y.

3.2. Preliminaries on Turaev's torsion function. The Euler characteristic of HF^+ is equal to Turaev's torsion function T. In this subsection we will briefly review the definition of T. The readers are referred to [19] if more details are desired.

Suppose Y is a closed oriented 3-manifold. The group $H = H_1(Y;\mathbb{Z})$ acts on $\mathrm{Spin}^c(Y)$. As in [11], we denote this action by addition. Fix a finite CW decomposition of Y, $\widetilde{Y}$ is the maximal abelian cover of Y with its induced CW structure. A family of cells (of all dimensions) in $\widetilde{Y}$ is said to be *fundamental* if over each cell of Y lies exactly one cell of this family. Choose a fundamental family of cells $\widetilde{e}$ in $\widetilde{Y}$, we get a basis for the cellular chain complex $C_*(\widetilde{Y})$ over the group ring $\mathbb{Z}[H]$. As shown in [19], $\widetilde{e}$ also gives rise to a Spin^c structure $\mathfrak{s} \in \mathrm{Spin}^c(Y)$.

Let $\mathbb{F}$ be a field and $\varphi\colon H \to \mathbb{F}^\times$ be a group homomorphism, $\mathfrak{s} \in \mathrm{Spin}^c(Y)$, and $\widetilde{e}$ be a fundamental family of cells which gives rise to $\mathfrak{s}$. Then one can define $\tau^\varphi(Y,\mathfrak{s}) \in \mathbb{F}$ to be the Reidemeister–Franz torsion of $C_*(\widetilde{Y})$ as in [19, Section 2.3].

Let $Q(H)$ be the classical ring of quotients of the group ring $\mathbb{Q}[H]$. $Q(H)$ splits as a finite direct sum of fields:

$$Q(H) = \oplus_{i=1}^n \mathbb{F}_i.$$

$\mathbb{F}_i$ is in the form $\mathbb{K}_i(t_1, t_2, \dots, t_b)$, where $\mathbb{K}_i$ is a cyclotomic field and $b = b_1(Y)$. Since $H \subset \mathbb{Q}[H] \subset Q(H)$, there are natural projections $\varphi_i\colon H \to \mathbb{F}_i$. Turaev defined

$$\tau(Y,\mathfrak{s}) = \sum_{i=1}^n \tau^{\varphi_i}(Y,\mathfrak{s}) \in \oplus_{i=1}^n \mathbb{F}_i = Q(H),$$

and showed that $\tau(Y,\mathfrak{s}) \in \mathbb{Z}[H]$ when $b_1(Y) \geq 2$. The coefficients of $\tau(Y,\mathfrak{s})$ gives the torsion function T. More precisely, when $b_1(Y) \geq 2$, T is defined by the following formula:

$$\tau(Y,\mathfrak{s}) = \sum_{h \in H} T(Y,\mathfrak{s} - h)h; \tag{2}$$

and when $b_1(Y) = 1$, one can define $T_t\colon \mathrm{Spin}^c(Y) \to \mathbb{Z}$ in a similar way, once a chamber t of $H_1(Y;\mathbb{R})$ is chosen.

Suppose the CW decomposition of Y has one 0-cell, m 1-cells, m 2-cells and one 3-cell. The chain complex $C_*(\widetilde{Y}) = (C_3 \to C_2 \to C_1 \to C_0)$, where C_0, C_1, C_2, C_3 are free $\mathbb{Z}[H]$-modules with ranks $1, m, m, 1$, respectively. Let $h_1, \dots, h_m$ be the generators of H represented by the 1-cells, and $g_1, \dots, g_m \in H$ be the elements which are dual to the 2-cells.

Denote by $\Delta_{r,s}$ the determinant of the matrix obtained from the $m \times m$-matrix (over $\mathbb{Z}[H]$) of the boundary homomorphism $C_2 \to C_1$ by deleting the rth row and sth column. Turaev proved the following equation (see (4.1.a) in [19]):

$$\tau(Y,\mathfrak{s})(g_r-1)(h_s-1)=\pm\Delta_{r,s}\in\mathbb{Z}[H]. \tag{3}$$

When $\mathfrak{s}$ is a nontorsion Spin^c structure, Ozsváth and Szabó showed in [12] that

$$\chi(HF^+(Y,\mathfrak{s}))=\pm T(Y,\mathfrak{s}). \tag{4}$$

3.3. Proof of the homological version. We deal with the case of $b_1(Y)\geq 2$ first. We will use the Heegaard splitting in Construction 3.2 as the fixed CW decomposition of Y. Each α_i corresponds to a 1-handle in Y, let a_i be the 1-cell which is the core of the 1-handle; each β_i corresponds to a 2-handle in Y, let b_i be the 2-cell which is the core of the 2-handle.

Let $\sigma\colon H\to\mathbb{Z}$ be the group homomorphism given by counting the intersection number with $[F]$. We construct the universal abelian cover $\widetilde{Y}$ of Y in two steps. First we take the infinite cyclic cover of Y dual to σ, denoted by Y^σ, which is the union of infinitely many copies of M:

$$Y^\sigma=\cdots\cup_{F_{-1}}M_{-1}\cup_{F_0}M_0\cup_{F_1}M_1\cup_{F_2}M_2\cup_{F_3}\cdots,$$

where F_0,F_1 are identified with F_-,F_+, respectively, if M_0 is identified with M. Then we take the cover $\pi\colon\ \widetilde{Y}\to Y^\sigma$.

We choose a lift of the 0-cell, a lift of the 3-cell, and $\widetilde{a}_i,\widetilde{b}_i$ which are lifts of the 1-cells and 2-cells. All lifts are chosen in $\pi^{-1}(M_0)$. This fundamental family of cells gives rise to a Spin^c structure $\mathfrak{s}\in\mathrm{Spin}^c(Y)$.

We extend the group homomorphism σ to a map of the group rings

$$\sigma\colon\ \mathbb{Z}[H]\to\mathbb{Z}[\mathbb{Z}]=\mathbb{Z}[q,q^{-1}].$$

By Equation (2), Equation (4) and the assumption that

$$HF^+(Y,[F],g-1)\cong HF^+(Y,[F],1-g)\cong\mathbb{Z},$$

we conclude that $\sigma(\tau(Y,\mathfrak{s}))$ is a Laurent polynomial of q with degree $2g-2$, and the coefficients of the highest term and the lowest term are ±1. Here the degree of a Laurent polynomial is defined to be the difference of the degree of the highest term and the degree of the lowest term.

In Equation (3), we choose $g_r\in H$ to be the element dual to the 2-cell corresponding to β_0, and h_s to be the element represented by the 1-cell corresponding to α_0. Note that if we cap off $\partial A_+=\alpha_0\cup\beta_0$, we get the surface F from A_+. We thus have $\sigma(g_r)=q^{\pm1},\sigma(h_s)=q^{\pm1}$. So σ(LHS of (3)) is a degree $2g$ Laurent polynomial with leading coefficient ±1. Here the leading coefficient of a Laurent polynomial is defined to be the coefficient of its lowest term.

Now we analyze the boundary map $\partial\colon C_2 \to C_1$. Suppose $1 \le i,j \le 2g + r$, and the coefficient of $\widetilde{a}_i$ in $\partial\widetilde{b}_j$ is $c_{ij} \in \mathbb{Z}[H]$. If $x \in \alpha_i \cap \beta_j$ lies in $A_\# \subset F_\#$, then x contributes a lift of a_i in $\pi^{-1}(M_0)$ to $\partial\widetilde{b}_j$ since $\widetilde{b}_j \subset \pi^{-1}(M_0)$; if $x \in \alpha_i \cap \beta_j$ lies in $A_+ \subset F_+$, then x contributes a lift of a_i in $\pi^{-1}(M_1)$ to $\partial\widetilde{b}_j$. Let $c^\#_{ij}$ (or c^+_{ij}) be the intersection number of α_i and β_j inside the domain $A_\#$ (or A_+), then we conclude that

$$\sigma(c_{ij}) = c^\#_{ij} + c^+_{ij}q.$$

If i or j is bigger than $2g$, then $c^+_{ij} = 0$.

Consider the matrix $C = (c_{ij})_{1\le i,j\le 2g+r}$. The result in the last paragraph implies that $\sigma(\det(C))$ is a polynomial of degree at most $2g$, and its constant term is $\det(c^\#_{ij})_{1\le i,j\le 2g+r}$. By (3), $\sigma(\det(C))$ is a degree $2g$ Laurent polynomial with leading coefficient ± 1. Hence

$$\det(c^\#_{ij})_{1\le i,j\le 2g+r} = \pm 1. \tag{5}$$

In Construction 3.2, let

$$N = M - \left(\mathrm{int}(D^+_b) \times [0,3]\right), \quad \gamma = (\partial D^+_b) \times [0,3].$$

(N,γ) is a sutured manifold. We claim that (N,γ) is a homology product, namely,

$$H_*(N, R_-(\gamma)) \cong H_*(N, R_+(\gamma)) \cong 0.$$

The proof is the same as [9, Proposition 3.1]. In fact, as in [9, Lemmas 3.2 and 3.3], using (5), one can show that $H_2(N;\mathbb{F}) = 0$ and the map

$$i_*\colon H_1(R_-(\gamma), \partial R_-(\gamma); \mathbb{F}) \to H_1(N,\gamma;\mathbb{F})$$

is injective for any field $\mathbb{F}$. Then the homological argument as in [9, Proposition 3.1] shows that (N,γ) is a homology product. Since M is obtained by capping off γ by $D^2 \times I$, M is also a homology product.

For the case of $b_1(Y) = 1$, the proof is essentially the same.

4. Characteristic product pairs. Using the surgery exact sequence and the adjunction inequality, one can prove the following result. Details of the proof can be found in [14, Lemma 5.4] and [8, Proposition 3.5, the second proof].

LEMMA 4.1. (Ozsváth–Szabó) *Suppose F is a closed connected surface in a closed manifold Y, and the genus of F is $g \ge 2$. Let Y' be the manifold obtained by cutting open Y along F and regluing by a self-homeomorphism of F. Let HF°*

denote one of $\widehat{HF}$ *and* HF^+. *Then we have*

$$HF^\circ(Y, [F], g-1) \cong HF^\circ(Y', [F], g-1).$$

We also need the following two simple lemmas.

LEMMA 4.2. *Suppose M is a compact 3-manifold with two boundary components F_-, F_+, and M is a homology product, namely,*

$$H_*(M, F_-) \cong H_*(M, F_+) \cong 0.$$

Suppose $F_0 \subset M$ is a closed surface that is homeomorphic to F_-, and F_0 splits M into two parts M_-, M_+, $\partial M_\pm = F_0 \cup F_\pm$. Then both M_- and M_+ are homology products.

Proof. From the exact sequence

$$\cdots \to H_1(M, F_-) \to H_1(M, M_-) \to H_0(M_-, F_-) \to \cdots$$

we conclude that $H_1(M, M_-) = 0$, thus the inclusion map $H_1(M_-) \to H_1(M)$ is surjective. Similarly, $H_1(M_+) \to H_1(M)$ is surjective.

Let g be the genus of F_-. Since M is a homology product, $H_2(M) \cong \mathbb{Z}$ is generated by $[F_-] = [F_+]$. So the maps $H_2(M_\pm) \to H_2(M)$ are surjective. We then have the short exact sequence

$$0 \to H_1(F_0) \to H_1(M_-) \oplus H_1(M_+) \to H_1(M) \to 0.$$

Both $H_1(M_-)$ and $H_1(M_+)$ surject onto $H_1(M) \cong H_1(F_-) \cong \mathbb{Z}^{2g} \cong H_1(F_0)$, so $H_1(M_-) \cong H_1(M_+) \cong \mathbb{Z}^{2g}$. It follows that the surjective maps $H_1(M_\pm) \to H_1(M)$ are actually isomorphisms. So we have the exact sequences

$$0 \to H_2(M_\pm) \to H_2(M) \to H_2(M, M_\pm) \to 0.$$

We already know that the maps $H_2(M_\pm) \to H_2(M)$ are surjective, hence $H_2(M, M_\pm) \cong 0$.

Now we have $H_*(M_-, F_0) \cong H_*(M, M_+) \cong 0$. The equality $H_*(M_-, F_-) \cong 0$ then follows from Poincaré duality and the Universal Coefficients Theorem. Hence M_- is a homology product, and so is M_+ by the same argument. □

The next lemma is well-known, proofs of it can be found in [7], [18].

LEMMA 4.3. *A homology class on a closed, orientable surface is represented by a simple closed curve if and only if it is primitive.*

Definition 4.4. Suppose S is a compact surface. The *norm* of S is defined by the formula:

$$x(S) = \sum_{S_i} \max\{0, -\chi(S_i)\},$$

where S_i runs over all the components of S.

The next theorem is an analogue of [10, Theorem 6.2′]. Here we just sketch the proof, and refer the readers to [9, Section 6] and [10] for more details.

THEOREM 4.5. *Suppose Y is a closed irreducible 3-manifold, $F \subset Y$ is a closed connected surface of genus $g \geq 2$. Suppose $\{F_1 = F, F_2, \dots, F_n\}$ is a maximal collection of mutually disjoint, nonparallel, genus g surfaces in the homology class of $[F]$. Cut open Y along $F_1, \dots, F_n$, we get n compact manifolds $M_1, \dots, M_n$, $\partial M_k = F_k \cup F_{k+1}$, where $F_{n+1} = F_1$. Let $\mathcal{E}_k$ be the subgroup of $H_1(M_k)$ spanned by the first homologies of the product annuli in M_k.*

If $HF^+(Y, [F], g-1) \cong \mathbb{Z}$, then for each k, $\mathcal{E}_k = H_1(M_k)$.

Sketch of proof. By Proposition 3.1 M is a homology product. Lemma 4.2 implies that each M_k is also a homology product.

Assume that $\mathcal{E}_k \neq H_1(M_k)$. By Lemma 4.3, we can find a simple closed curve $\omega \subset F_k$, such that $[\omega] \notin \mathcal{E}_k$. Let $\omega_- = \omega \subset F_k$. Since M_k is a homology product, by Lemma 4.3 there is a simple closed curve $\omega_+ \subset F_{k+1}$ which is homologous to ω in M_k. We fix an arc $\sigma \subset M_k$ connecting F_k to F_{k+1}. Let $\mathcal{S}_m(+\omega)$ be the set of properly embedded surfaces $S \subset M_k$, such that $\partial S = \omega_- \sqcup (-\omega_+)$, and the algebraic intersection number of S with σ is m. Here $-\omega_+$ denotes the curve ω_+, but with opposite orientation. Similarly, let $\mathcal{S}_m(-\omega)$ be the set of properly embedded surfaces $S \subset M_k$, such that $\partial S = (-\omega_-) \sqcup \omega_+$, and the algebraic intersection number of S with σ is m. Since M_k is a homology product, $\mathcal{S}_m(\pm\omega) \neq \emptyset$. Let $x(\mathcal{S}_m(\pm\omega))$ be the minimal value of $x(S)$ for all $S \in \mathcal{S}_m(\pm\omega)$, where $x(S)$ is the norm of S.

Claim. For positive integers p, q,

$$x(\mathcal{S}_p(+\omega)) + x(\mathcal{S}_q(-\omega)) > (p+q)x(F_k). \tag{6}$$

Proof of Claim. Suppose $S_1 \in \mathcal{S}_p(+\omega), S_2 \in \mathcal{S}_q(-\omega)$. Isotope S_1, S_2 so that they are transverse, then perform oriented cut-and-paste to S_1, S_2, we get a closed surface $P \subset \mathrm{int}(M_k)$, with $x(P) = x(S_1) + x(S_2)$. Using standard arguments in 3-dimensional topology, we can assume P has no component which is a sphere or torus.

Since M is a homology product, one can glue the two boundary components of M together to get a manifold Z, which has the same homology as $F \times S^1$, so

$H_2(Z) \cong H_1(F) \oplus H_2(F)$. Thus if a closed surface $H \subset Z$ is disjoint from one F_k, then H must be homologous to a multiple of F.

P is homologous to $(p+q)F_k$ in Z; in fact, as shown in [9], P is the disjoint union of $p+q$ surfaces $P_1, \dots, P_{p+q}$, where each P_i is homologous to F_k. Since $HF^+(Z, [F], g-1) \neq 0$, F is Thurston norm minimizing in Z. So we have $x(P_i) \geq x(F) = x(F_k)$. So if $x(P) \leq (p+q)x(F_k)$, then the equality holds, and each P_i has $x(P_i) = x(F_k)$.

Next we claim that P_i has only one component. Otherwise, suppose $P_i = Q_1 \sqcup Q_2$, then

$$x(Q_1), x(Q_2) < x(P_i) = x(F_k).$$

$[Q_1], [Q_2]$ are multiples of $[F_k]$ in $H_2(Z)$. Since F_k is Thurston norm minimizing in Z and $[F_k]$ is primitive, we must have $[Q_1] = [Q_2] = 0$, which is impossible.

Since $\{F_1, \dots, F_n\}$ is a maximal collection of disjoint, nonparallel, genus g surfaces, each P_i is parallel to either F_k or F_{k+1}. Thus there exists $r \in \{0, 1, \dots, p+q\}$, such that $P_1, \dots, P_r$ are parallel to F_k, $P_{r+1}, \dots, P_{p+q}$ are parallel to F_{k+1}. Let $C_r = P_r \cap S_1$. Then $C_r \times I$ is a collection of annuli which connect P_r to P_{r+1}, while C_r is homologous to ω. This contradicts the assumption that $[\omega] \notin \mathcal{E}_k$. Now the proof of the claim is finished. □

As shown in [9, Lemma 6.4], when m is sufficiently large, there exist connected surfaces $S_1 \in \mathcal{S}_m(+\omega)$ and $S_2 \in \mathcal{S}_m(-\omega)$, such that they give taut decompositions of M_k. By the work of Gabai [2], one can construct two taut smooth foliations $\mathscr{F}'_1, \mathscr{F}'_2$ of M_k, such that F_k, F_{k+1} are leaves of the two foliations; one can also construct a taut smooth foliation $\mathscr{F}$ of $Z - \mathrm{int}(M_k)$ with compact leaves F_k, F_{k+1}. Let $\mathscr{F}_i = \mathscr{F}'_i \cup \mathscr{F}$ be a foliation of Z. Let R be a connected surface in Z, whose intersection with F_k is ω. As in [4] or [9], using (6), one can prove that

$$\langle c_1(\mathscr{F}_1), [R] \rangle \neq \langle c_1(\mathscr{F}_2), [R] \rangle.$$

Thus [4, Theorem 3.8] can be applied to show that $\mathrm{rank}(HF^+(Y, [F], g-1)) > 1$, a contradiction. This finishes the proof of Theorem 4.5. □

COROLLARY 4.6. *Suppose* (Π_k, Ψ_k) *is the characteristic product pair [10, Definition 6] for* M_k*, then the map*

$$i_*\colon H_1(\Pi_k) \to H_1(M_k)$$

is surjective.

Proof. The proof is the same as [10, Corollary 7]. □

LEMMA 4.7. *Notation is as above, then each* Π_k *contains a product manifold* $G_k \times I$*, where* G_k *is a once-punctured torus.*

Proof. This is an easy consequence of Corollary 4.6. In fact, let $E_k = \Pi_k \cap F_k$, $E_k^c = \overline{F_k - E_k}$, we can construct a graph Γ as follows. The vertices of Γ correspond to the components of E_k and E_k^c. $E_k \cap E_k^c$ consists of simple closed curves. For each of such curves, we draw an edge connecting the two components of E_k and E_k^c that are adjacent along the curve. No component of $E_k \cap E_k^c$ is a nonseparating curve in F_k, otherwise there would be a closed curve $c \subset F_k$ which intersects the component exactly once, thus $[c] \notin H_1(\Pi_k)$. It then follows that the Γ contains no loop, so Γ is a tree.

Consider a root of the tree, it corresponds to a component H of E_k or E_k^c. H has only one boundary component since it corresponds to a root. H is not a disk, so it contains a once-punctured torus G_k. Since $H_1(G_k)$ contributes to $H_1(M_k)$ nontrivially, H must be a component of E_k. □

5. Proof of the main theorem. In this section, we will use Heegaard Floer homology with twisted coefficients to prove Theorem 1.1.

LEMMA 5.1. *Suppose Z is a closed 3-manifold containing a nonseparating two-sphere S, $K \subset Z$ is a null-homologous knot, H is a genus $g(>0)$ Seifert surface for K. Let $Z_0(K)$ be the manifold obtained by doing 0-surgery on K, $\widehat{H}$ be the extension of H in $Z_0(K)$. Let $\omega \subset Z - K$ be a closed curve such that $\omega \cdot S \neq 0$. We then have*

$$\underline{\widehat{HFK}}(Z, K, \omega, [H], g; \mathcal{L}) \cong \underline{HF}^+(Z_0(K), \omega, [\widehat{H}], g-1; \mathcal{L}).$$

Proof. As in [13, Corollary 4.5], when p is sufficiently large, we have two exact triangles: (we suppress $[H], [\widehat{H}]$ and $\mathcal{L}$ in the notation)

$$\cdots \to \underline{\widehat{HFK}}(Z, K, \omega, g) \xrightarrow{\sigma} \underline{HF}^+(Z_p, \omega, [g-1]) \to \underline{HF}^+(Z, \omega) \to \cdots,$$

$$\cdots \to \underline{HF}^+(Z_0, \omega, [g-1]) \xrightarrow{\sigma'} \underline{HF}^+(Z_p, \omega, [g-1]) \to \underline{HF}^+(Z, \omega) \to \cdots.$$

By Lemma 2.1, $\underline{HF}^+(Z, \omega) = 0$, so the maps σ, σ' are isomorphisms, hence our desired result holds. □

Proof of Theorem 1.1. Notation is as in Section 4. By Lemma 4.7, we have the product manifolds $G_k \times I \subset M_k$. By cut-and-reglue along F_k's, we can get a new manifold Y_1 such that the $G_k \times I$'s are matched together to form an essential submanifold $G \times S^1$ in Y_1, where G is a once-punctured torus in F.

Since each M_k is a homology product, we can construct a new manifold Y_2 with $b_1(Y_2) = 1$ by cutting Y_1 open along F and then regluing by a homeomorphism of F.

By Lemma 4.1,

$$HF^+(Y_1, [F], g-1) \cong HF^+(Y_2, [F], g-1) \cong HF^+(Y, [F], g-1) \cong \mathbb{Z}.$$

Let $D \subset G$ be a small disk. We remove $D \times S^1$ from Y_1, then glue in a solid torus V, such that the meridian of V is $p \times S^1$ for a point $p \in \partial D$. The new manifold is denoted by Z, and the core of V is a null-homologous knot K in Z. $\check{F} = F - \mathrm{int}(D)$ is a Seifert surface for K. Conversely, Y_1 can be obtained from Z by 0-surgery on K.

Z contains nonseparating spheres. In fact, pick any properly embedded nonseparating arc $c \subset G - \mathrm{int}(D)$, such that $\partial c \subset \partial D$, then $\partial(c \times S^1)$ bounds two disks in V. The union of the two disks and $c \times S^1$ is a nonseparating sphere in Z. Suppose S is such a nonseparating sphere. Let $\omega \subset Z - \check{F}$ be a closed curve such that $\omega \cdot S \neq 0$. The curve ω can also be viewed as lying in Y_1 and Y_2, and ω is disjoint from F.

Since $b_1(Y_2) = 1$ and $\omega \cdot [F] = 0$, ω is null-homologous in Y_2, so

$$\underline{HF}^+(Y_2, \omega, [F], g-1; \mathcal{L}) \cong HF^+(Y_2, [F], g-1; \mathbb{Q}) \otimes_{\mathbb{Q}} \mathcal{L} \cong \mathcal{L}.$$

A twisted version of Lemma 4.1 then implies that

$$\underline{HF}^+(Y_1, \omega, [F], g-1; \mathcal{L}) \cong \mathcal{L}.$$

By Lemma 5.1, $\underline{\widehat{HFK}}(Z, K, \omega, [\check{F}], g; \mathcal{L}) \cong \underline{HF}^+(Y_1, \omega, [F], g-1; \mathcal{L}) \cong \mathcal{L}$. Now by Theorem 2.2 K is fibred with fibre $\check{F}$, hence Y_1 is fibred with fibre F, and so is Y. □

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