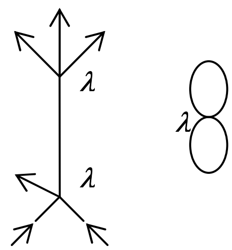
Ph225a **Problem Set #2 (Chapter II.3. and II.4.)** October 11, 2004

1. (a) Draw all the diagrams describing two mesons producing four mesons with second order interactions λ^2. Write down the corresponding amplitudes of the Feynman diagrams. [*Hint*: The diagrams must consist of 6 external sources and 2 vertices, and therefore only $(6 + 4 \times 2)/2 = 7$ total lines are allowed.]

   (b) Now consider interactions in the third order λ^3 for two mesons producing four mesons. One of the diagrams is shown below, which includes a connected "tree" diagram and a disconnected "bubble" diagram. Draw all other third-order (λ^3) diagrams associated with this given tree having a bubble connected to one of its lines. Write down the corresponding amplitude for any one of the connected tree-and-bubble diagrams. [*Hint*: Can you figure out that 9 total lines and therefore only 3 internal lines are allowed for the connected tree-and-bubble diagrams?] [*N.B.*: For diagrams with disconnected bubbles, which you need not consider for this problem, the number of internal lines can exceed 3.]



2. In this problem we'll derive the energy and momentum conservation laws for actions that observe the Lorentz symmetry. We define the space-time coordinate x^μ and the energy-momentum four-vector p^μ:

$$x^\mu \equiv (x^0, x^i) = (t, \vec{x}), \qquad p^\mu \equiv (p^0, p^i) = (E, \vec{p}),$$

where t denotes the time coordinate, and E is the energy. Now consider a translation $x^\mu \to x^\mu + a^\mu$, where a^μ is a constant, with a^0 representing time displacements and a^i representing space displacements. Under such displacements, a scalar field $\varphi(x)$ transforms as $\varphi(x) \to \varphi(x+a)$, and $\delta\varphi(x) = \varphi(x+a) - \varphi(x) = a^\mu \partial_\mu \varphi(x)$. Similarly, the variation of the Lagrangian L is given by $\delta L = a^\mu \partial_\mu L$.

   (a) Prove that the energy-momentum tensor defined as

$$T_\nu^\mu \equiv \frac{\delta L}{\delta \partial_\mu \varphi} \partial_\nu \varphi - \delta_\nu^\mu L$$

   is a conserved quantity under the translation. That is, $\partial_\mu T_\nu^\mu = 0$.

   (b) The energy-momentum four-vector p^μ is related to the energy-momentum tensor via the definition

$$p^\mu \equiv \int d^3x \, T_0^\mu .$$

   Using the conservation law of T_ν^μ, show that the energy-momentum conservation law $dp^\mu / dt = 0$ holds under the translation. In addition, for the Klein-Gordon action, show that $T_0^{\ 0}$ denotes the energy density.

   (c) The energy-momentum tensor defined above is not a measurable physical quantity because of an ambiguity associated with its definition. We can exploit this ambiguity by adding to the energy-momentum tensor a term $\partial_\lambda E^{\lambda\mu\nu}$, where $E^{\lambda\mu\nu}$ is anti-symmetric in the first two indices. That is $E^{\lambda\mu\nu} = -E^{\mu\lambda\nu}$. Show that under the replacement $T^{\mu\nu} \to T^{\mu\nu} + \partial_\lambda E^{\lambda\mu\nu}$, $T^{\mu\nu}$ is still conserved. Also show that the energy-momentum conservation law still holds with the addition of a term $\partial_\lambda E^{\lambda\mu\nu}$, as long as the tensor $T^{\mu\nu}$ vanishes sufficiently rapidly at infinity. We can therefore choose an energy-momentum tensor $T^{\mu\nu}$ in such a way that it is symmetric in μ and ν.