

Introduction

What kind of waves have you heard of?

Sound wave, water wave, light wave, electromagnetic wave, seismic wave, gravitational wave ...

Wave as a form of motion is ubiquitous. Moreover, understanding waves is a key step to go from classical mechanics to quantum mechanics. That is why we are having a whole course on waves.

Syllabus

In this course, we will study the generation, propagation, and various properties of waves.

- Simple harmonic oscillation: one degree of freedom; Forced oscillation and resonance
- Normal modes: simple harmonic oscillation of more than one degrees of freedom
- Waves: free and forced oscillation in an infinite system
- Traveling waves; Standing waves
- Reflection and transmission
- Modulations, pulses, and wave packets
- Waves in two and three dimensions: Polarization
- Interference and Diffraction

Logistics

- Instructor: Xie Chen, Office: 163 W. Bridge, Mail Code: 149-33, Phone: x3793, Email: xiechen@caltech.edu. Office hour: every Wednesday, 4-5pm at 163 W Bridge.
- TA: Charles Xu, cxu3@caltech.edu; Jinrui Hou, jinruihou@caltech.edu
- Grader: Michael Yao, myao@caltech.edu
- Lectures: Tuesdays 10:30 am - 11:55 am, 269 Lauritsen; Thursdays 10:30 am - 11:55 am, 201 E Bridge.
- Prerequisites: Math 1abc or equivalent (differential equations, matrices, trigonometry, complex numbers, etc.); Physics 1abc, or equivalent (mechanics, electromagnetism, etc.); Mathematics (basics).

- Recommended textbooks (not required):
 - Georgi, The Physics of Waves (can be downloaded from <http://www.people.fas.harvard.edu/hgeorgi/new.htm>)
 - Crawford, Waves (can be downloaded from https://archive.org/details/Waves_371)
 - French, Vibrations and Waves (on reserve at the library)
 - The Feynman Lectures, chap 21-36, 47-51
(can be viewed online at <http://www.feynmanlectures.caltech.edu/>)
 - Mathematica training sessions <http://url.wolfram.com/2FLjmnPC/>

- Course website

<http://www.its.caltech.edu/~xcchen/courses/physics12a.html>

- Problem Sets

Posted on the website every Thursday, due on the following Thursday by 5pm. Please put finished homework in a wooden box marked ‘Ph12a Inbox’ in Bridge Annex. Solutions will be posted also on the website. Graded problem sets can be picked up in a mail slot marked ‘Ph12a Outbox’ in Bridge Annex one week after the due date.

Each problem set contains a ‘Questions and Comments’ section, from which I hope to gather your feedback about the class. You can ask about things covered or not covered in the lecture; comments and suggestions about the class will be greatly appreciated. I will try to address the questions in class. For the ones that I don’t have time to talk about, the TAs will discuss them in the recitation sessions.

- Exams

Final exam in the last week of the term.

- Grading

60% problem sets, 40% final exam.

- Extension policy

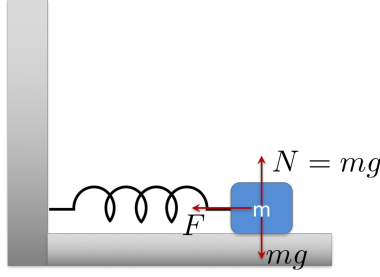
One silver bullet for one week extension (with proper excuse). After that, 50% credit for late homework up to one week; no credit after that. Please communicate beforehand with the grader about extension.

1 Simple Harmonic Oscillator

To study waves, we start from its most elementary components: a simple harmonic oscillator (SHO). A SHO is a small periodic motion of one degree of freedom. It can be realized in many different forms.

1.1 Example I: mass on a spring

Consider the small oscillation of a mass attached to a massless spring on a frictionless surface. The one degree of freedom is x – the position of the mass. The mass experiences three forces: the



gravitational force mg which is balanced by the supporting force from the surface $N = mg$ and the restoring force from the spring F . Therefore, the motion $x(t)$ of the mass is only affected by the restoring force F . Suppose that x_0 is the position of m when the spring is relaxed, then Hooke's law says

$$F = -k(x - x_0) \quad (1)$$

where k is the spring constant. Notice the minus sign in this formula, indicating that the force is a restoring force.

Without loss of generality, let's set $x_0 = 0$. We have $F = -kx$. Combined with Newton's law $F = ma = m\frac{d^2x}{dt^2}$, we get the equation of motion

$$m\frac{d^2x}{dt^2} = -kx \quad (2)$$

To solve this equation, let's try solutions of the form $x(t) = A \cos(\omega t) + B \sin(\omega t)$, which contains three unknown variables A , B and ω . Taking the first derivative with respect to time, we get

$$\frac{dx}{dt} = -A\omega \sin(\omega t) + B\omega \cos(\omega t) \quad (3)$$

Taking derivative again, we get

$$\frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t) - B\omega^2 \sin(\omega t) \quad (4)$$

Plugging this back to the equation of motion we find

$$-m\omega^2(A \cos(\omega t) + B \sin(\omega t)) = -k(A \cos(\omega t) + B \sin(\omega t)) \quad (5)$$

Hence

$$\omega = \sqrt{\frac{k}{m}} \quad (6)$$

ω is called the angular frequency of the oscillation.

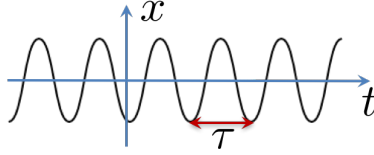
The equation of motion alone cannot determine A and B . They are related to the initial conditions of the oscillation. From the form of the solution we can see that

$$x(t=0) = A, \quad x'(t=0) = B\omega \quad (7)$$

Therefore, the general form of the solution is given by

$$x(t) = x(0) \cos(\omega t) + \frac{1}{\omega} x'(0) \sin(\omega t) \quad (8)$$

which graphically looks like



The time it takes for the motion to repeat itself is called a period, which we denote by τ , is equal to $\frac{2\pi}{\omega}$. We have

$$x(t + \tau) = x(t), \quad x'(t + \tau) = x'(t) \quad (9)$$

The frequency of the oscillation ν is given by $\nu = \frac{1}{\tau}$, which is related to the angular frequency as $\omega = 2\pi\nu$.

We can in general put $x(t)$ into the form

$$x(t) = C \sin(\omega t + \phi) \quad (10)$$

where $C = \sqrt{(x(0))^2 + \frac{(x'(0))^2}{\omega^2}}$ is called the amplitude of the oscillation and ϕ , determined from $\sin \phi = x(0)/C$, $\cos \phi = x'(0)/(\omega C)$, is called the phase of the oscillation.