Physics 12a **Assignment 9** **Caltech, 12/05/19**

Web Page: http://www.its.caltech.edu/~xcchen/courses/physics12a.html

Problems: You don't need to turn in this assignment. It is only to help you study for the final.

1.) Wave plate and polaroid I

A plane wave of light traveling in the $+z$ direction is polarized at an angle θ from the x axis in the $x - y$ plane. When it encounters a sheet of polaroid in the $z = L$ plane that transmits only light polarized at an angle $\theta + \frac{\pi}{2}$, the wave is completely absorbed. However, if the plane wave first passes through a sheet of cellophane in the $z = 0$ plane with the 'fast axis' along the x axis, some of the light gets through. Suppose that the cellophane introduces a phase difference of ϕ between the component of the light wave polarized along the fast (x) axis and the component polarized along the slow (y) axis. Find the ratio of the intensity of the transmitted wave beyond the polaroid to the incoming wave intensity as a function of θ and ϕ. Hint: does your answer go to zero as $\phi \to 0$? What happens as $\theta \to 0$?

2.) Wave plate and polaroid II

A plane wave of light traveling in the $+z$ direction is polarized in the x direction. When it encounters a sheet of polaroid in the $z = L$ plane that transmits only y polarized light, the wave is completely absorbed. However, if the plane wave first passes through a sheet of cellophane in the $z = 0$ plane with the 'fast axis' at an angle θ with the x axis, some of the light get through. Suppose that the cellophane introduces a phase difference of ϕ between a wave polarized along the fast axis and one polarized along the slow axis. Find the ratio of the intensity of the transmitted wave beyond the polaroid to the incoming wave intensity as a function of θ and ϕ.

Compare the result with the previous problem and explain what is going on.

3.) Amplitude modulation and nonlinearity.

a) One way to produce an amplitude-modulated carrier wave is to pass a current $I = I_0 \cos \omega_0 t$ oscillating at the carrier frequency ω_0 through a resistance R which is not constant but has a component that varies at the modulation frequency ω_m, that is, $R = R_0(l + a_m \cos \omega_m t)$. (In a "carbon-granule" microphone the resistance is modulated by the motion of a diaphragm, which compresses the carbon granules that provide the resistance.) The voltage $V = IR$ across the resistor is an amplitude-modulated carrier wave. Find the expression for V in terms of a superposition of carrier (frequency w_0), upper sideband (frequency $\omega_0 + \omega_m$), and lower sideband (frequency $\omega_0 - \omega_m$).

b) Alternatively, suppose we happen to start with two voltages, one oscillating at the carrier frequency, the other at the modulation frequency. The problem is this: How can you physically combine these two voltages, $V_0 = A_0 \cos \omega_0 t$ and $V_m = A_m \cos \omega_m t$ in such a way as to produce an amplitude-modulated carrier wave? First, suppose you merely superpose the two voltages, i.e., you put them both on the broadcasting antenna. Will this work?

c) Next, suppose that the voltages in part (b), after being superposed, are then applied to the input of a voltage amplifier. (For example they may be applied between control grid and cathode of a radio tube.) Suppose that the amplifier is a linear amplifier, i.e., its output (for example the plate-to-cathode voltage of the tube) is proportional to the input. Will this work?

d) Finally, suppose that the amplifier output has both a linear and a quadratic component, as follows:

$$V_{out} = A_1 V_{in} + A_2 (V_{in})^2.$$

Let $V_{in} = V_0 + V_m$ as defined in part (b). Show that because of the nonlinear quadratic term $A_2(V_{in})^2$ the amplifier output includes, among other things, an amplitude-modulated carrier wave, with modulation amplitude proportional to A_m.

4.) Group and phase velocity.

The group velocity of the peak of a wave packet is $v_g = d\omega(k)/dk$, evaluated at the central wavenumber k characterizing the packet. Show that for light in a medium with refraction index $n(\lambda)$,

$$\frac{1}{v_g} = \frac{1}{v_\phi} - \frac{1}{c}\lambda \frac{dn(\lambda)}{d\lambda},$$

where λ is the vacuum wavelength of the light.