Physics 12a **Assignment 8** **Caltech, 11/21/19**

Web Page: http://www.its.caltech.edu/∼xcchen/courses/physics12a.html

Problems: Due in the Ph12 IN box in Bridge Annex, 5:00pm, Thursday, 12/05/19.

1.) Reflections in transmission lines.

In the last homework, we derived the ratio between the transmitted wave, the reflected wave and the incoming wave. The amplitude of the voltage of the three waves are related as

$$\begin{cases} \frac{A_r^U}{A_i^U} = \frac{z_R - z_L}{z_L + z_R} \\ \frac{A_t^U}{A_i^U} = \frac{2z_R}{z_L + z_R} \end{cases} \tag{1}$$

Use this formula to do the following problem.

(Note that at the same time, the amplitude of the current of the three waves are related as

$$\begin{cases} \frac{A_r^I}{A_i^I} = \frac{z_L - z_R}{z_L + z_R} \\ \frac{A_t^I}{A_i^I} = \frac{2z_L}{z_L + z_R} \end{cases} \tag{2}$$

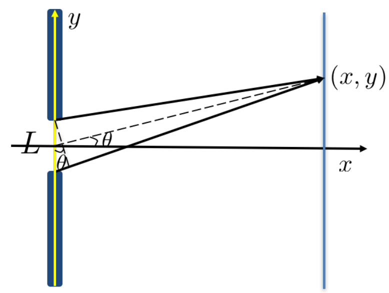
)

Suppose a parallel plate transmission line having 50 ohms impedance is joined to one having 100 ohms impedance.

- (a) A voltage pulse of +10 volts (maximum value) is incident from the 50 ohm line to the 100 ohm line. What is the "height" (in volts, including the sign) of the reflected pulse? Of the transmitted pulse?
- (b) A + 10-volt pulse is incident from the 100 ohm to the 50 ohm line. What are the reflected and transmitted pulse heights?
- (c) How can you insert an ordinary resistor so that an incident pulse traveling from the 50 ohm to the 100 ohm line is transmitted without generating any reflected pulse? You need to specify how many ohms the resistance has, and give a schematic sketch showing the two plates of each of the transmission lines at the place where they join and showing the resistor connected.
- (d) Suppose a + 10-volt pulse is incident on this modified circuit. What is the size of the transmitted pulse?
- (e) Now suppose a +10-volt pulse is sent down this line in the "wrong" direction, i.e., from the 100 ohm line to the 50 ohm line. What happens? Find the reflected and transmitted pulse heights.
- (f) Next consider the problem of transmitting a pulse from the 100 ohm line to the 50 ohm line without generating any reflection. What should be the resistance value and how should it be connected at the place where the lines are joined?

  Hint: Consider a semi-infinite parallel plate as shown in the figure.

  From the point of view of point A and B, the parallel plate acts in exactly the same way as a normal resistor: the voltage across AB is proportional to the current that flows from A to B and their ratio is the impedance (resistance). So in the above problem, when an extra resistor is connected to the parallel plates, the total impedance (resistance) can be calculated using the usual formula for connected resistors.



2.) Single Slit Diffraction.

Consider the diffraction pattern of a single slit with width L. In the lecture, we derived the intensity of the diffraction pattern to be

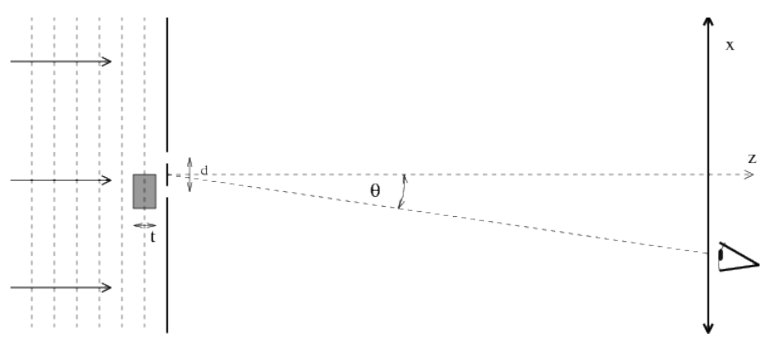
$$I \propto \frac{\sin^2\left(\frac{kL}{2x}y\right)}{y^2} \tag{3}$$

So the intensity becomes zero when $y = \frac{nx\lambda}{L}$, $n \neq 0$. Let's see if we can make sense of this fact.

(1) Using the figure above, show that when $y = \frac{nx\lambda}{L}$, $n \neq 0$, the distance from different points in the slits to the observing point (x, y) changes by integer multiples of wavelength from the bottom to the top of the slit.

(2) Explain why the intensity becomes zero at these locations. (Ignore the change in the amplitude of the wave emanating from different points in the slit.)

3.) Double Slit Interference. Two small slits in the $z = 0$ plane spaced d apart are illuminated by a plane wave with wavenumber k in the $+z$ direction. As shown in the figure, the light



heading towards the *lower* slit must pass through a block of material of thickness t and index of refraction $n > 1$. The index of refraction is defined as the ratio of the wave number in the material and that in the surrounding environment.

$$n = \frac{k'}{k} \tag{4}$$

An observer a *long* distance away measures the light intensity as a function of the angle θ ($\theta << 1$). Assume that the slits are infinitely small (that is we ignore the diffraction effect of the slits).

a) What is the phase difference of the wave at the two slits?

b) Calculate the observed time-averaged intensity distribution as a function of the angle θ. Consider only the case when θ is small ($<< 1$). (Neglect any absorption or reflection off of the faces of the block of material. Also you can ignore the difference in amplitude of the two waves coming from the two slits.)

c) Sketch by hand the interference patterns with and without the block, and describe *qualitatively* the difference(s) between the two cases. (Describe how the interference pattern changes as n increases from 1.)

d) Is the intensity distribution the same for $x < 0$ as for $x > 0$? Under what conditions for n and t *is* the distribution the same?

4.) Four slits.

An opaque screen with four infinitely narrow slits at $x = \pm 0.6$ mm and $x = \pm 0.4$ mm is blocking a beam of monochromatic light with wavelength 4×10^{-5} cm.

(a) Derive the formula for the interference pattern that appears on a screen 5 meters away. Plot it using Mathematica (or other softwares you like).

(b) Now make the slits to be 0.04 mm wide. Derive the formula for the interference pattern. Plot it using Mathematica (or other softwares you like).

5.) Transmission Diffraction Grating.

Consider a transmission diffraction grating with a large number of slits, each of which has a width of L and are separated from each other by a distance of a. The grating is illuminated by a plane wave of monochromatic light that is traveling in a direction perpendicular to the grating.

(1) When $L << a$, sketch by hand the resulting intensity pattern when observed at a large distance away from the grating.

(2) When $a = 2L$, sketch by hand the resulting intensity pattern when observed at a large distance away from the grating.

For both questions, you don't need to be very accurate about all the details of the pattern. It is sufficient to identify the main peaks. Ignore the difference in the amplitude of the wave coming from different slits.

6.) Questions and Comments.