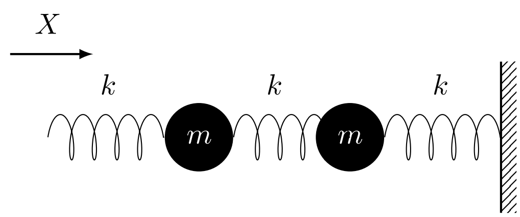
Physics 12a **Assignment 5** **Caltech, 10/31/19**

Web Page: http://www.its.caltech.edu/∼xcchen/courses/physics12a.html

Problems: Due in the Ph12 IN box in Bridge Annex, 5:00pm, Thursday, 11/7/19.

1) This problem is the same as problem 3 of the last homework but we are going to do it in a different way.

Two equal masses m are connected to three identical springs (spring constant k) on a frictionless horizontal surface (see figure). The relaxed length of the springs is a. One end of the system is fixed; the other is driven back and forth with a displacement $X = X_0 \cos(\omega t)$. Find the resulting displacements of the two masses.



(1) Instead of writing the EOM for the two masses directly, consider them as part of an infinite system of identical masses coupled to identical springs. What is the EOM of the infinite system? What are the traveling wave eigen modes?

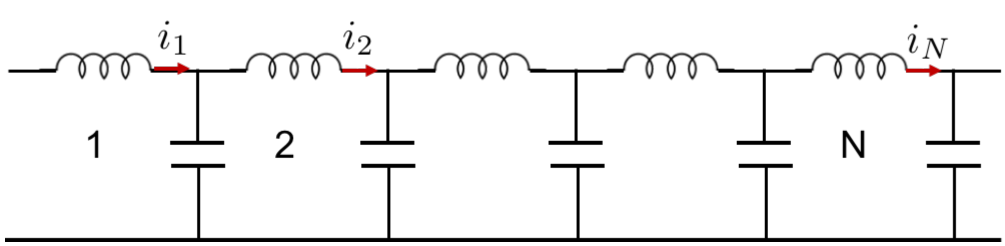
(2) The original system can be thought of as a four body subsystem of the infinite system with the boundary condition that $x_0 = X_0 \cos(\omega t)$, $x_3 = 0$. In the steady state, i.e. assume small friction so that the transient modes all decay to zero, which traveling wave modes of the infinite system do we potentially need to satisfy this boundary condition ?

(3) Make a superposition of the corresponding traveling wave modes $\psi(x,t) = \sum_j |\alpha^{(j)}| \cos(\omega^{(j)} t + k^{(j)} x + \varphi^{(j)})$. Determine $|\alpha^{(j)}|$ and $\varphi^{(j)}$ from the boundary condition.

(4) Find x_1 and x_2. Does this match your result from last homework (assuming small friction so that the transient modes all decay to zero)? (If you cannot do the comparison because you do not have your previous homework at hand, do not worry. The comparison part will not count towards the total grade.)

2) Translation invariant LC circuit

Consider a translation invariant LC circuit as shown in the following figure. The Nth unit is connected to the first unit in a periodic way.



Let's set up the equation of motion and find the traveling wave in this system.

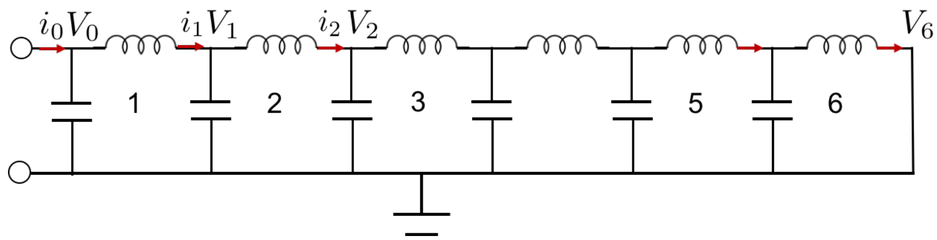
(1) Choose the degree of freedom to be the current passing through the nth inductor, i_n. Write down the equation of motion for each unit using the law that the voltage drop around each unit has to be zero.

(2) Write the equation of motion in the matrix form $\frac{d^2}{dt^2}I = KI$, where $I = \begin{pmatrix} i_1 \\ \vdots \\ i_N \end{pmatrix}$. Find the matrix K. (Hint: It should look very similar to the mass on the spring system.)

(3) Solve the eigen equation for K and find the traveling wave modes of this circuit.

3) LC circuit with boundary condition.

Consider the following LC network circuit:



All the capacitors have the same capacitance C, and all the inductors have the same inductance L and no resistance. The bottom wire is grounded (has zero voltage). When we apply a harmonically oscillating signal from a signal generator through a coaxial cable to V_0, different oscillating voltages will be induced along the line in the steady state. That is if $V_0(t) = V\cos\omega t$, then $V_j(t)$ has the form $V_j(t) = A_j \cos\omega t + B_j \sin\omega t$.

We are going to consider this system as a translation invariant LC circuit with boundary condition.

(1) Now consider the boundary condition shown in the above figure. Given that $V_0(t) = V\cos\omega t$, what kind of boundary condition does this impose on $i_0 - i_1$?

(2) Given that $V_6(t) = 0$, what kind of boundary condition does this impose on $i_6 - i_7$?

(3) Assume that the driven oscillation has a very long wave length. Write the boundary condition as constrains on the space derivative of the current at the end points. For a fixed ω, how many different traveling wave modes are there? Make a superposition of these modes, with amplitude and phase shift of each mode as free parameter.

(4) Use the two boundary conditions to determine the free parameters. You may need to use the following trigonometry formula

$$\begin{aligned} \cos(\alpha+\beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta), \\ \cos(\alpha-\beta) &= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta), \\ \sin(\alpha+\beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta), \\ \sin(\alpha-\beta) &= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta) \end{aligned} \tag{1}$$

4) Questions and Comments.