Physics 12a **Assignment 4** **Caltech, 10/24/19**

Web Page: http://www.its.caltech.edu/∼xcchen/courses/physics12a.html

Problems: Due in the Ph12 LOCK box in Bridge Annex, 5pm, Thursday, 10/31/19.

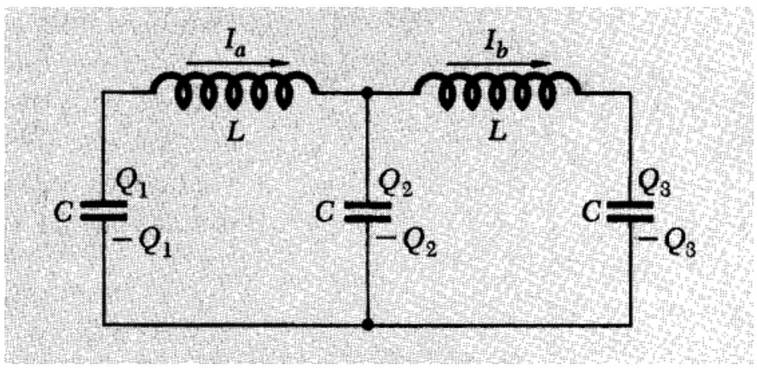
Mathematica: You can use Mathematica or other computing software to diagonalize matrices.

1) Oscillations of two coupled LC circuits.

Consider the coupled LC circuit as shown in the figure.

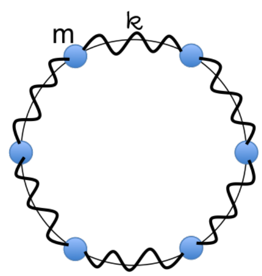
(a) Use I_a and I_b as degrees of freedom, write down the equations of motion.

(b) Find the normal modes of oscillation in this system.



2) Coupled oscillators with translation symmetry.

Consider a system of six masses coupled with springs along a ring. The masses and springs are all the same and the system is translation invariant.



(1) The equation of motion can be written in the form $\frac{d^2}{dt^2}X = -\frac{1}{m}KX$. What is the matrix K?

(2) Diagonalize K using Mathematica. Find all the eigenvalues and eigenvectors.

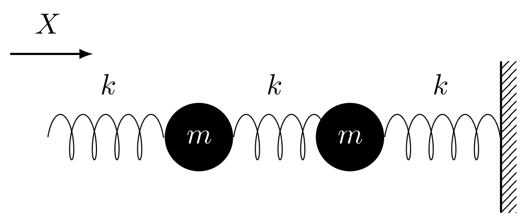
(3) Write down the matrix S which represents translation symmetry in the system. Can you find a set of eigenvectors A of K such that they transform under S as $SA = \beta A$ where β is a complex number? (Hint: you may have to take linear superpositions of the degenerate eigenvectors found by Mathematica. Take a pair of degenerate eigenmodes A_1, A_2, make a superposition of them $A = a_1 A_1 + a_2 A_2$ such that A satisfy $SA = \beta A$. Use this condition to determine β and the relation between a_1 and a_2.)

(4) Now suppose the masses also interact with next nearest neighbors through springs of a different Hooke's constant. What is the new form of K?

(5) Diagonalize the new K using Mathematica. Can you still find a set of eigenvectors A such that $SA = \beta A$? What are the corresponding eigen frequencies? Do the eigenvectors change? Do the eigen frequencies change?

3) Coupled oscillator with boundary condition.

Two equal masses m are connected to three identical springs (spring constant k) on a frictionless horizontal surface (see figure). One end of the system is fixed; the other is driven back and forth with a displacement $X = X_0 \cos(\omega t)$.



(a) Write down the equation of motion for the two masses. Write the equation in the matrix form and as an inhomogeneous second order differential equation.

(b) When the the left end is fixed, what are the eigen modes of the system? Find the eigen frequency and describe the motion of each mass in the eigen modes.

(c) When the left end is driven back and forth with a displacement $X = X_0 \cos(\omega t)$, find the steady state of the system. That is, suppose that the two masses oscillate with the same frequency as the driving force, find their amplitude and relative phase with respect to the driving force. Under what condition is the amplitude infinite? Why?

(We are going to solve the same problem using a different method in the next homework.)

4) Questions and Comments