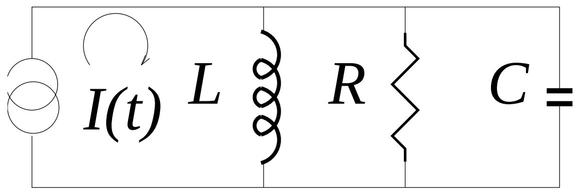
Web Page: http://www.its.caltech.edu/∼xcchen/courses/physics12a.html

Problems: Due in the Ph12 IN box in Bridge Annex, 5pm, Thursday, 10/24/19.

Mathematica: You can use Mathematica or other computing software to diagonalize matrices.

1) Resonance in parallel RLC circuits: A resistor R, a capacitor C, and an inductor L are all driven in *parallel* with a current $I(t) = I_0 \cos \omega t$.

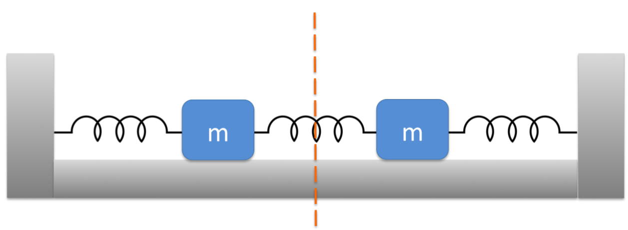


Find:
(a) the equation of motion governing the system;
(b) the resonant frequency ω_0;
(c) the FWHM of the resonance peak Γ and quality factor $Q = \omega_0/\Gamma$;
(d) the average power absorbed at resonance. Hint: the power absorbed equals the product between the total output current $I(t)$ and the voltage $V(t)$ across the system.

(We consider the situation where $\Gamma << \omega_0$, so that the resonance peak in terms of power absorption and the resonance peak in terms of amplitude match each other.)

2) Coupled oscillators with reflection symmetry.

Consider the configuration as shown in the following figure where two blocks with the same mass are connected to three springs with the same Hooke's constant which are all relaxed at equilibrium position.



(1) Write down the equation of motion. Follow the steps we took in class, turn the EOM into an eigen equation of matrix $N = M^{-1}K$.

(2) The system is reflection symmetric. How does reflection symmetry act as a two by two matrix R on the two degree of freedom x_1 and x_2 (displacement of the two masses)? Show that $RN = NR$.

(3) Find the eigenvectors of N which satisfy $NA = \omega^2 A$. Find the corresponding oscillation eigenmodes (that is, write down $x_1(t)$ and $x_2(t)$ in each mode).

(4) How do the eigenvectors transform under reflection? (That is, apply R to A and find how A changes.)

(5) Now consider the situation where the middle spring has a different Hooke's constant than the other two. The system is still reflection symmetric. Using symmetry argument, show that the eigenvectors, hence the eigenmodes, of the system do not change. Can the frequency of the eigenmodes change?

3) Normal modes of a CO_2 molecule.

The CO_2 molecule can be crudely and classically modeled as a system with a central mass $m_2 = 12$ AMU connected by equal springs of spring constant k to two masses $m_1 = m_3 = 16$ AMU, constrained to move only along the line joining their centers.

(1) Set up and solve the equations for the two normal modes in which the masses oscillate along that line.

(2) There are three masses but only two oscillatory normal modes. What's the third mode?

(3) What is the ratio between the frequencies of the two normal modes of oscillation?

4) Questions and Comments