Physics 12a **Assignment 1** **Caltech, 10/03/19**

Problems: Due in the Ph12a IN box in Bridge Annex, 5:00pm, Thursday, 10/10/19.

I.1) Mass on a vertical spring
A massless spring with no mass attached to it hangs from the ceiling. Its length is 20 cm. A mass M is now hung on the lower end of the spring. Support the mass with your hand so that the spring remains relaxed, then suddenly remove your supporting hand. The mass and spring oscillate. The lowest position of the mass during the oscillation is 10 cm below the place it was resting when you supported it.

a. What is the frequency of oscillation?

b. What is the velocity when the mass is 5 cm below its original resting place?

A second mass of 300 g is added to the first mass, making a total of $M + 300$ g. When this system oscillates, it has half the frequency of the system with mass M alone.

c. What is M?

d. Where is the new equilibrium position?

I.2) The superposition principle for inhomogenous linear equations of motion. If a simple harmonic oscillator with displacement $\psi(t)$ is driven by an external driving force $F(t)$ which does not depend on $\psi(t)$, the equation of motion,

$$M\frac{d^2\psi(t)}{dt^2} = -K\psi(t) + F(t),$$

is linear but *in*homogeneous. Prove the superposition principle: if a driving force $F_1(t)$ produces an oscillation $\psi_1(t)$ (when F_1 is the only driving force), and if another driving force $F_2(t)$ produces and oscillation $\psi_2(t)$ (when F_2 is the only driving force), then if both driving forces are present simultaneously, the corresponding oscillation is given by the superposition $\psi(t) = \psi_1(t) + \psi_2(t)$.

I.3) Linear differential equations.
Find the solutions to the linear differential equation $d^3f(t)/dt^3 + f(t) = 0$ using the trial function $f(t) = f_0e^{st}$. s is in general a complex number. Show that if $f(t)$ is a solution, then both the real part of $f(t)$ and the imaginary part of $f(t)$ are solutions of the equation.

I.4) Questions and Comments