1. Consider the cyclic group $C_n$.

   - If $n$ is the product of two distinct primes $p_1$ and $p_2$, is $C_n$ isomorphic to $C_{p_1} \times C_{p_2}$? Explain your answer.
   - If $n$ is the square of a prime $p$, is $C_n$ isomorphic to $C_p \times C_p$? Explain your answer.

2. Consider the quaternion group $Q = gp\{x, y\}, x^4 = e, x^2 = y^2, y^{-1}xy = x^{-1}$.

   Find a faithful matrix representation of $Q$. (hint: the simplest one is two dimensional. In a faithful representation, different group elements are mapped to different matrices.)

3. Given a normal subgroup $N$ of a group $G$, a representation $D^{G/N}$ of the quotient group $G/N$ can be lifted to give a representation $D^G$ of the full group $G$ by the following definition:

   \[ D^G(g) := D^{G/N}(gN) \] (1)

   That is, each element of the group is assigned the matrix $D^{G/N}$ of the coset to which it belongs.

   - Verify that $D^G(g)$ indeed provides a representation of $G$, i.e. $D^G(g_1)D^G(g_2) = D^G(g_1g_2)$.
   - What is the kernel of this representation?

4. Find $n$ different one dimensional representations of the cyclic group $C_n$. Verify that they are orthogonal to each other. Are there higher dimensional irreducible representations of $C_n$?