Physics 129b **Homework 3** **Due 01/30/20 by 5pm**

1. Consider the cyclic group C_n.

- If n is the product of two distinct primes p_1 and p_2, is C_n isomorphic to $C_{p_1} \times C_{p_2}$? Explain your answer.
- If n is the square of a prime p, is C_n isomorphic to $C_p \times C_p$? Explain your answer.

2. Consider the quaternion group $Q = gp\{x, y\}, x^4 = e, x^2 = y^2, y^{-1}xy = x^{-1}$.

Find a faithful matrix representation of Q. (hint: the simplest one is two dimensional. In a faithful representation, different group elements are mapped to different matrices.)

3. Given a normal subgroup N of a group G, a representation $D^{G/N}$ of the quotient group G/N can be lifted to give a representation D^G of the full group G by the following definition:

$$D^G(g) := D^{G/N}(gN) \tag{1}$$

That is, each element of the group is assigned the matrix $D^{G/N}$ of the coset to which it belongs.

- Verify that $D^G(g)$ indeed provides a representation of G, i.e. $D^G(g_1)D^G(g_2) = D^G(g_1g_2)$.
- What is the kernel of this representation?

4. Find n different one dimensional representations of the cyclic group C_n. Verify that they are orthogonal to each other. Are there higher dimensional irreducible representations of C_n?