Fundamental limits of distributed tracking

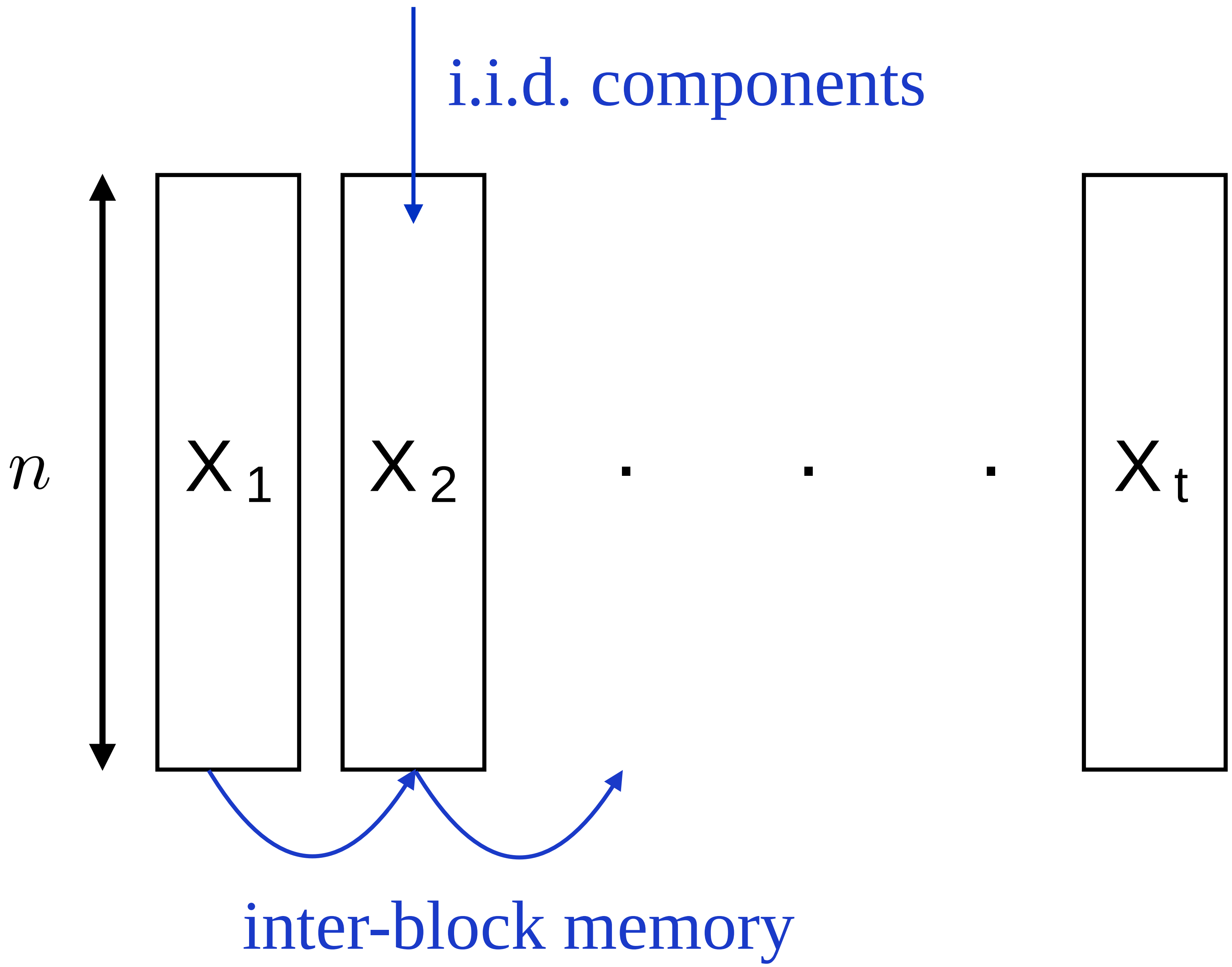
Victoria Kostina

Babak Hassibi

Caltech

ISIT 2020

Class of sources



- no restrictions on the source alphabet

$t = 1$: Classical CEO problem

Chief Estimation Officer, or Chief Executive Officer

- Introduced (discrete-alphabet source) by Berger-Zhang-Viswanathan’96.
- Viswanathan-Berger’97: an achievability bound on the rate-distortion dimension of the symmetric Gaussian CEO problem.
- Oohama’98: sum rate - distortion region for the symmetric Gaussian CEO problem.
- Prabhakaran-Tse-Ramanchandran’04: full Gaussian CEO rate region.
- Chen-Zhang-Berger-Wicker’04: water-filling representation of the minimum sum rate.
- Chen-Berger’08, Behroozi-Soleymani’09: successive coding schemes.
- Wagner-Tavildar-Viswanath’08: found the rate region of the distributed Gaussian lossy compression problem by coupling it to the CEO problem.
- Wang-Chen-Wu’10: a simple converse on the sum rate.
- Courtade-Weissman’14: the rate-distortion regions of the distributed source coding and the CEO problem under logarithmic loss.

Sum rate - distortion function

Definition

An (R, d, t) causal CEO code: K causal encoders operating at sum rate R and a causal decoder that achieve distortion d over time horizon t.

Definition

The rate-distortion pair (R, d) is achievable at time horizon t in the CEO problem if $\gamma > 0$, n_0 N such that $n \geq n_0$, an $(R, d + \gamma, t)$ exists. The causal sum rate - distortion function is defined as follows:

$$R_{\mathrm{CEO}\,t}(d) \,\square\, \inf \overset{\square}{} R\colon (R, d) \text{ is achievable at time horizon } t \overset{\square}{}$$

Notation

- $X_{[t]} \triangleq (X_1, \ldots, X_t)$
- $U_{[t]}^{k} \triangleq (U_1^k, \ldots, U_t^k)$ ⟵ codewords by k-th encoder up to time t
- $U_{[t]}^{[K]} \triangleq \begin{bmatrix} U_{[t]}^{1} \\ \vdots \\ U_{[t]}^{K} \end{bmatrix}$
- Massey's directed information ('90)

$$I(X_{[t]} \to \hat{X}_{[t]}) \triangleq \sum_{i=1}^{t} I(X_{[i]}; \hat{X}_i | \hat{X}_{[i-1]})$$

- Causally conditional (Kramer, '98) probability kernel

$$P_{Y_{[t]}||X_{[t]}} \triangleq \prod_{i=1}^{t} P_{Y_i|Y_{[i-1]}, X_{[i]}}$$

Berger-Tung achievability bound with inter-block memory

$$R_{t\,\mathrm{CEO}}(d) \le \inf_{\substack{P_{\mathsf{U}_{[t]}^{[K]} \| \mathsf{Y}_{[t]}^{[K]}} = \prod_{k=1}^{K} P_{\mathsf{U}_{[t]}^{k} \| \mathsf{Y}_{[t]}^{k}}, \\ P_{\hat{\mathsf{X}}_{[t]} \| \mathsf{U}_{[t]}^{[K]}} : \\ \frac{1}{t}\sum_{i=1}^{t} \mathbb{E}\left[\mathsf{d}\left(\mathsf{X}_i, \hat{\mathsf{X}}_i\right)\right] \le d}} I\left(\mathsf{Y}_{[t]}^{[K]} \to \mathsf{U}_{[t]}^{[K]}\right)$$

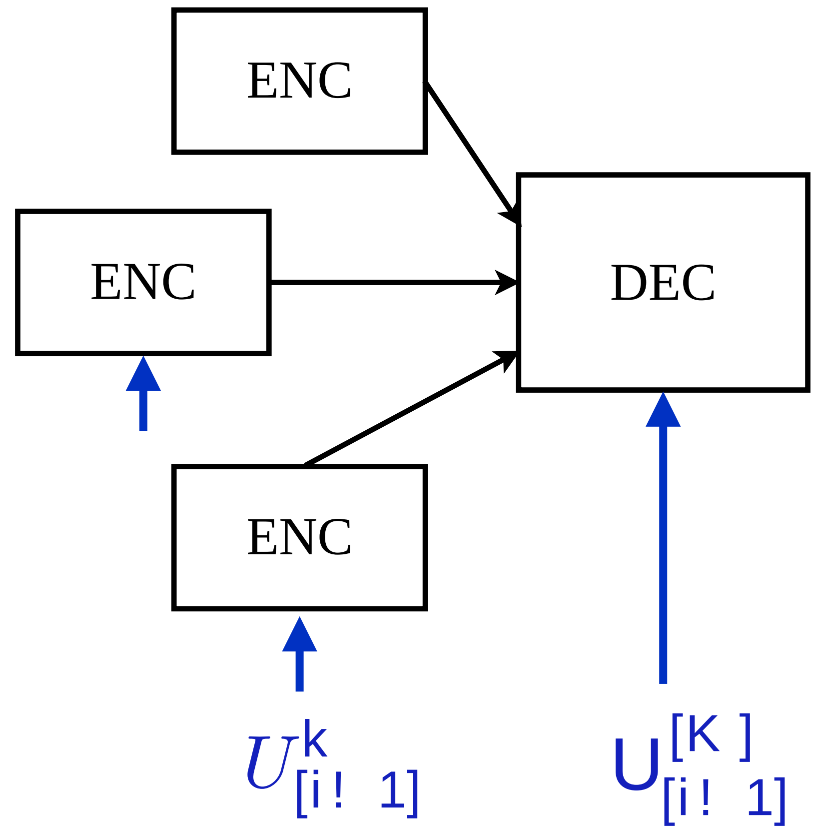
single-letter

Converse

- Data processing
- Chain rule
- Standard single-letterization argument

Achievability

Step i :



Coded SI

need to consider steps $\{1, 2, \ldots, t\}$ jointly

Structured multiterminal source coding problem with $K \cdot t$ encoders and t decoders

Nonasymptotic Berger-Tung achievability bound with inter-block memory

- We make use of the achievability proof technique developed by Yassaee, Aref, Gohari, 2013.
- It uses a stochastic likelihood coder.
- The technique extends naturally to causal coding problems.
- We extend Yassaee et al. '13 to $K > 2$ by proposing a novel decoder that falls into the class of generalized likelihood decoders (Merhav '17).

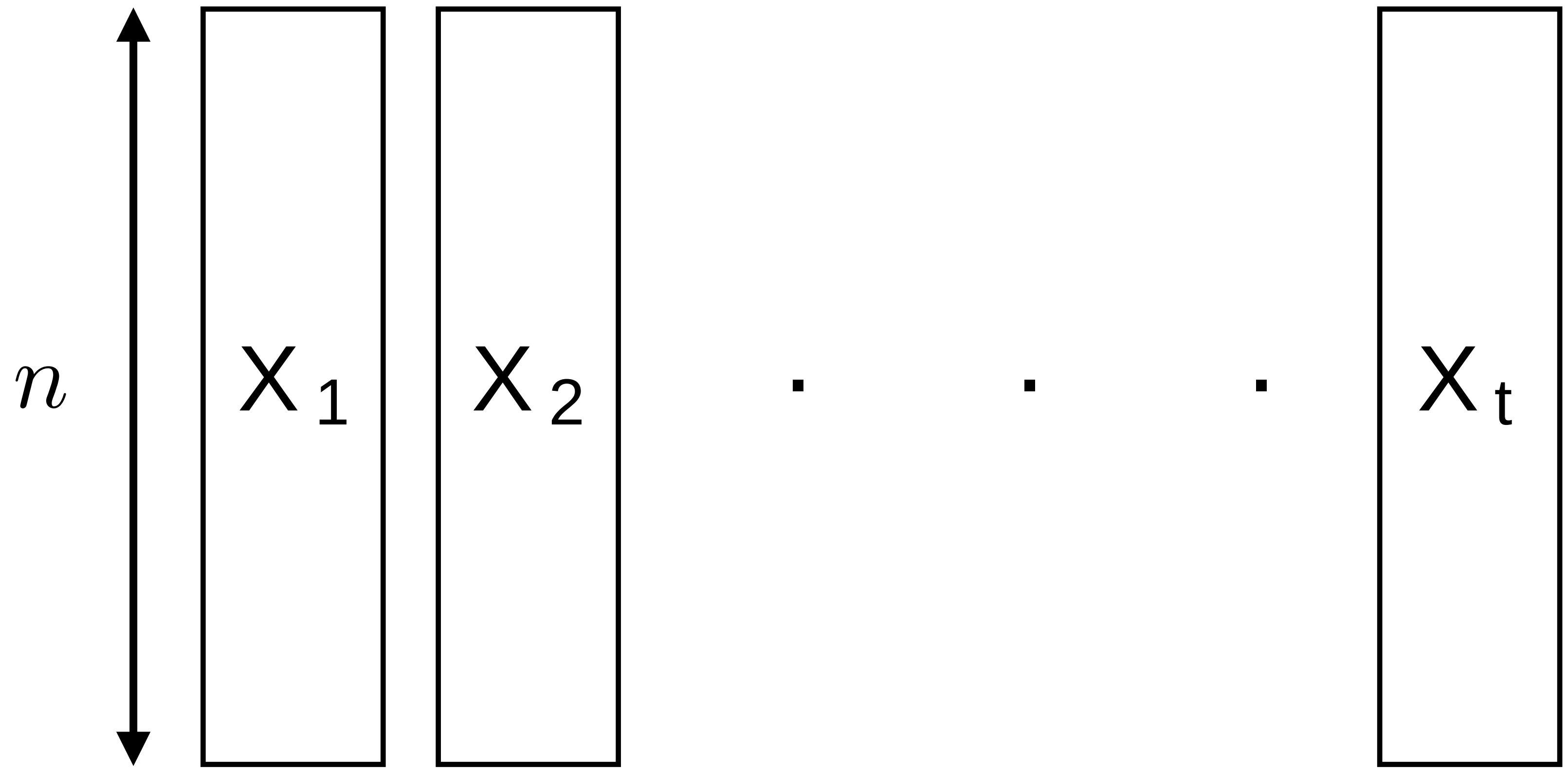
Two parts

1. General achievability and converse bounds.
2. Explicit fundamental tradeoff in the Gaussian case.

Gauss-Markov source: a simple source with memory

$$X_1, \{V_i\}_{i=1}^{\infty} \sim \mathcal{N}\left(0, \sigma^2 \mathsf{I}_n\right) \text{ i.i.d.}$$

$$X_{i+1} = aX_i + V_i$$



- $a = 0$: i.i.d. Gaussian source
- $|a| < 1$: asymptotically stationary source
- $|a| \geq 1$: nonstationary source

Gauss-Markov source: a simple source with memory

$$X_{i+1} = aX_i + V_i$$

¥ P. Tian and V. Kostina, "The dispersion of the Gauss-Markov source", IEEE Transactions on Information Theory, 2019.

¥ P. Tian and V. Kostina, "From parameter estimation to dispersion of nonstati
Gauss-Markov processes", arXiv:1907.00304, 2019.

¥ V. Kostina and B. Hassibi, "Rate-cost tradeoffs in control", IEEE Transactions on Automatic Control, 2019.

¥ V. Kostina and B. Hassibi, "Rate-cost tradeoffs in scalar LQG control and tra
with side information", Allerton, 2018.

¥ A. Khina, A. Khisti, V. Kostina, and B. Hassibi, "Tracking and control of Gaus
Markov processes over packet-drop channels with acknowledgements", IEEE Transactions on Control of Network Systems, 2019.