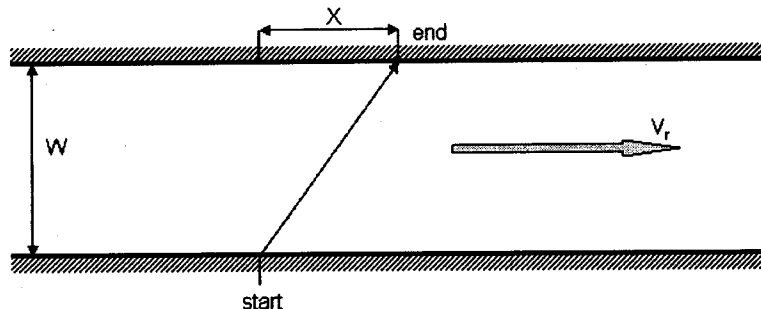


## Swimming Lesson



You decide to swim across a river. The current in the river has speed  $V_r$ , left to right as seen from your point of view on the riverbank. The river has a constant width  $W$ .

You want to arrive on the opposite bank a distance  $X$  from the point directly across from your starting point. Note that  $X$  may be positive or negative; call  $X$  positive if you end up downstream (i.e. to the right) of your starting point, negative if you manage to end up upstream (i.e. to the left).

Let  $V_s$  be your swimming speed, measured relative to the water, and assume you swim in a straight line.

Note: You may use vectors to solve this problem, but it is not necessary.

- (2 points) (a) The trip takes a time  $T$ . How far did you swim, in the frame of reference of the water? Express your answer two ways, once in terms of  $T$  and  $V_s$  and again in terms of  $W$ ,  $X$ ,  $V_r$ , and  $T$ .
- (1 point) (b) At what angle are you moving, in the frame of reference of the water? A formula for the tangent of this angle, will suffice. (Define the angle so that zero means perpendicular to the river banks and pointed to the opposite shore. Let positive angles denote downstream swimming, negative angles upstream swimming.)
- (1 point) (c) What is the minimum swimming speed  $V_s$  you must attain to reach the opposite bank at  $X = 0$ ? (i.e. directly opposite your starting point)
- (2 points) (d) Using the results of part (a), find an equation, quadratic in  $T$ , which relates  $T$ ,  $X$ ,  $W$ ,  $V_s$  and  $V_r$ . Solve for  $T$ .
- (1 point) (e) Assuming you want to arrive at  $X = 0$ , and have sufficient speed, how long will the trip take?
- (3 points) (f) Your aim is to arrive on the opposite bank in the least amount of time. What is this minimum time and what value of  $X$  corresponds to it? In the reference frame of the water, what is the angle at which you are swimming?