Ph1a Quiz Solution 1

Fall 2009

1.1 Problem 1 (6 points)

1.1.a (2 points)

Assume the boat rises up on its skis at time t_1.

For $t < t_1$ the acceleration $a = 0.20\,\text{m/s}^2$ is constant.

So the velocity $v(t) = v(0) + at = 0 + at = at$. In particular $v(t_1) = at_1 = 20\,\text{km/hr} = 5.56\,\text{m/s}$, so $t_1 = 27.8$ sec.

For $t < t_1$ the displacement $x(t) = \frac{1}{2}(v(0) + v(t))t$, so $x(t_1) = \frac{1}{2}(0 + v(t_1))t_1 = \frac{1}{2}(0 + 5.56) \cdot 27.8 = 77.2\text{m}$.

1.1.b (2 points)

Assume the boat get to 1km at time $t_1 + t_2$.

For $t_1 < t < t_1 + t_2$ the acceleration $a' = 0.40\,\text{m/s}^2$ is constant.

So $v(t) = v(t_1) + a'(t - t_1)$ and $x(t) = x(t_1) + \frac{1}{2}(v(t_1) + v(t))(t - t_1)$.

In particular when time $t = t_1 + t_2$, $v(t_1 + t_2) = 5.56 + a'(t_2)$, $x(t_1 + t_2) = 77.2 + \frac{1}{2}(v(t_1) + v(t_1 + t_2))t_2 = \frac{1}{2}(5.56 + 5.56 + a't_2)t_2 = 77.2 + 5.56t_2 + 0.20t_2^2 = 1000\text{m}$

This leads to solving $0.20t_2^2 + 5.56t_2 - 922.8 = 0$ and taking the positive solution, so $t_2 = 55.4\text{sec}$, $t_1 + t_2 = 83.2\text{sec}$ and that that time, $v(t_1 + t_2) = 5.56 + a'(t_2) = 5.56 + 0.40 \cdot 55.4 = 27.7\text{m/s}$.

1.1.c (2 points)

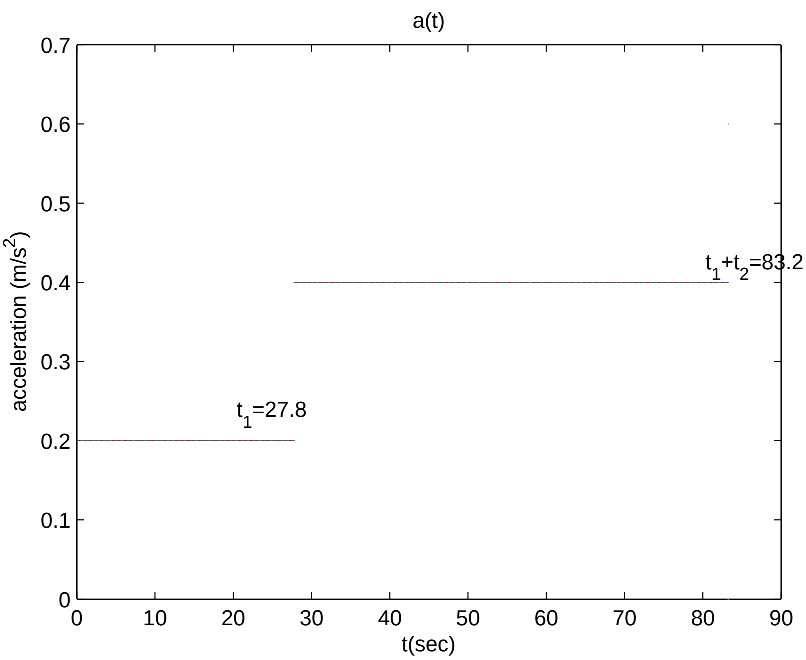

Figure 1: a(t)

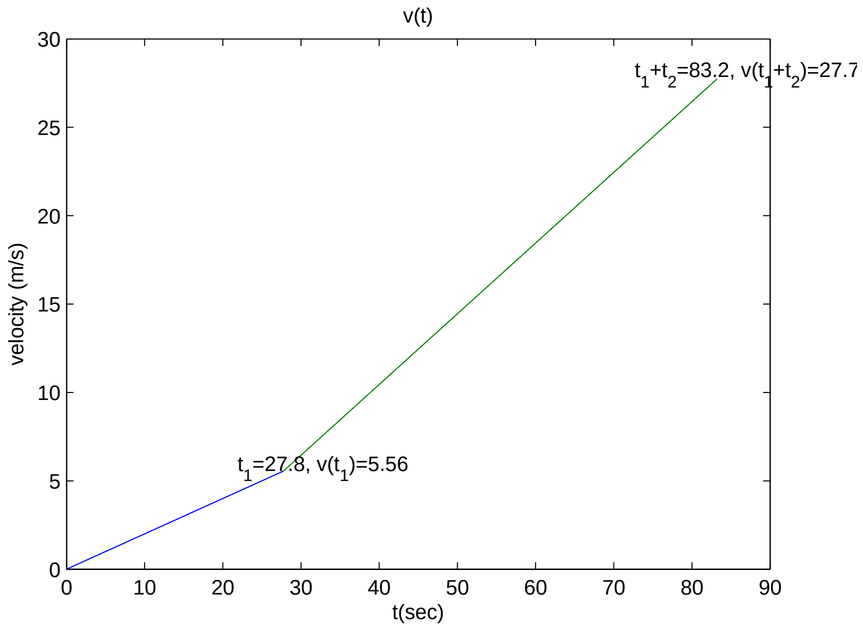

Figure 2: v(t)

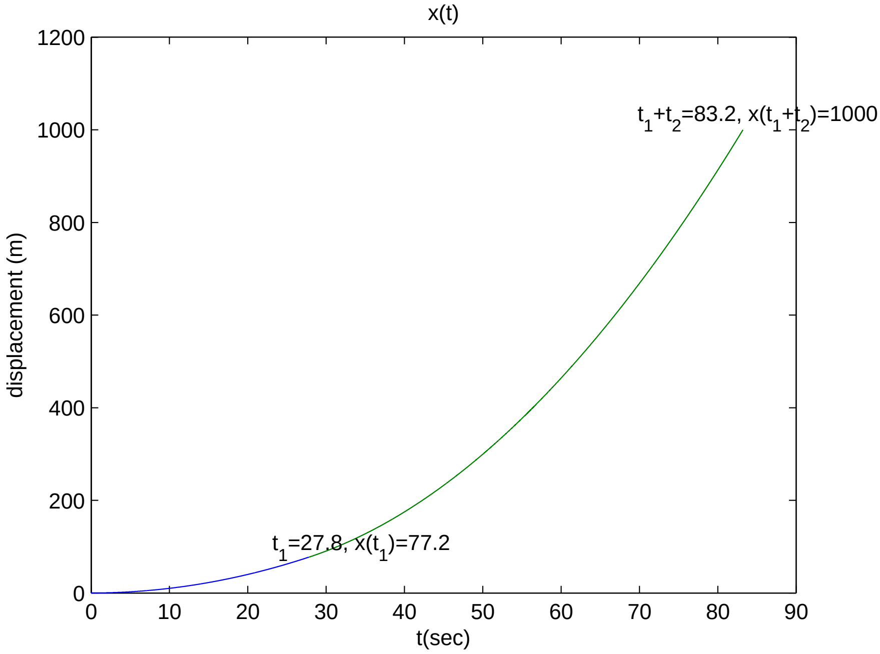

Figure 3: x(t)

1.2 Problem 2 (4 points)

1.2.a (2 points)

After released, the weights begin to fall freely. There's no tension on the strings.

For $i \geq 2$, the height of weight i is $H_i = L_1 + L_2 + ... + L_{i-1}$ in the beginning. The time before its landing is t_i.

So $\frac{1}{2}gt_i^2 = H_i$, $t_i = \sqrt{\frac{2}{g}H_i} = \sqrt{\frac{2}{g}(L_1 + L_2 + ... + L_{i-1})}$

1.2.b (2 points)

Since the time intervals between successive landings are all equal to the time between when the string is released and when weight 2 reaches the ground, which is t_2.

We have $t_i = (i-1)t_2$. $\sqrt{\frac{2}{g}(L_1 + L_2 + ... + L_{i-1})} = (i-1)\sqrt{\frac{2}{g}L_1}$.

Square both sides and get $L_1 + L_2 + ... + L_{i-1} = (i-1)^2 L_1$. For all $i \geq 2$. Also $L_1 + L_2 + ... + L_{i-1} + L_i = i^2 L_1$. Subtracting these two equations and get $L_i = (2i-1)L_1$.

The last weight has $i = n-1$, so $L_{n-1} = (2n-3)L_1$.