Problem 2 (6 points) - Marble in a Bowl

A marble of mass $m$ is deposited inside a hemispherical bowl of radius $R$, as shown in the figure. The bowl is then spun around its vertical axis with a constant angular velocity $\omega$. The marble eventually settles at a distance $r$ from the bowl’s vertical axis while rotating around that axis with the same angular velocity $\omega$ as the bowl itself.

(a) (3 points) Find the force that the bowl exerts on the marble. Give the total magnitude of the force as well as its angle with respect to the vertical axis of the bowl.

(b) (2 points) Derive an expression for $r$ in terms of $R$, $\omega$, and the gravitational acceleration $g$.

Note that, for small enough $\omega$, the answer to part (b) does not make sense.

(c) (1 point) Explain what happens physically when the angular velocity $\omega$ is too small.