Problem 1 (6 points)
A uniform disk of mass $m$ and radius $r$ begins to slide down an inclined plane with an initial velocity $v_0$ at its center of mass at time $t = 0$. The inclined plane has a surface frictional coefficient $\mu$ and forms an angle $\theta$ relative to the ground, as shown in Figure 1A. At time $t = t_1$, the disk begins to roll down the plane without slipping. The local gravitational acceleration is $g$, pointing vertically down.

(a) (2 points) Express $t_1$ in terms of $v_0$, $g$, $\mu$ and $\theta$.

(b) (1 point) Find the minimal frictional coefficient $\mu$ (in terms of $g$ and $\theta$) required for the disk to achieve pure rolling motion.

At $t > t_1$ the disk reaches the end of the inclined plane with a final speed $v_f$ at its center of mass, and it becomes stuck instantaneously upon impact to the end of a uniform thin rod of length $L$ and mass $M$ hanging vertically from the ceiling. The rod-disk assembly swings to the right, as shown in Figure 1B.

(c) (1 point) Find the moment of inertia $I$ of the rod-disk assembly about the axis through the pivot.

(d) (2 points) Find the angular momentum (both the magnitude and direction) of the rod-disk assembly about the axis through the pivot after the impact. Express your answers in terms of $v_f$, $m$, $M$, $r$ and $L$. Discuss the condition required for the rod-disk assembly to swing to the right.