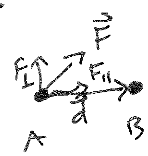
Phys 1a Lecture 9 10/26/11

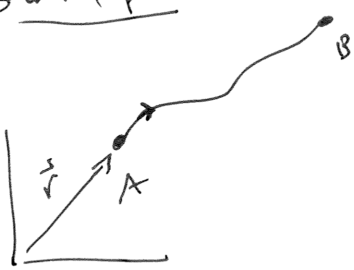
- Energy
  - work
  - kinetic
  - potential
- Conservation
- Power

Work



$$W = (F_\parallel)(d)$$
$$= \vec{F}\cdot\vec{d}$$

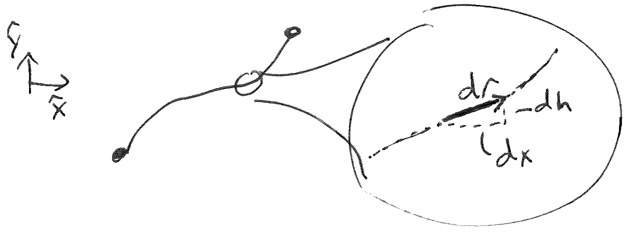
General path



$$W_{A\to B} = \int_A^B \vec{F}\cdot d\vec{r} = \int_{t_A}^{t_B} \vec{F}(\vec{r}(t))\cdot\frac{d\vec{r}}{dt}\,dt$$

Example $\vec{F}$: gravity on Earth

$$\vec{F} = -mg\hat{y}$$



$$W_{A\to B} = \int_A^B (-mg\hat{y})\cdot(dx\,\hat{x} + dh\,\hat{y})$$
$$= -mg\int_A^B dh$$
$$= -mgh_B + mgh_A$$

Define: $U(h) \equiv \underbrace{mgh}_{} + \text{const}$ → "potential energy"

$$\Rightarrow\ W_{A\to B} = U(h_A) - U(h_B)$$

And now, a generic look:

$$W_{A\to B} = \int_A^B \vec{F}\cdot d\vec{r} = \int_A^B m\left(\frac{d\vec{v}}{dt}\cdot\vec{v}\right)dt$$

$\vec{F}$ — $m\vec{a}$ — $m\dfrac{d\vec{v}}{dt}$

$d\vec{r}$ — $\dfrac{d\vec{r}}{dt}\,dt$ — $\vec{v}\,dt$

Aside:

$$\frac{d}{dt}(\vec{v}\cdot\vec{v}) = 2\frac{d\vec{v}}{dt}\cdot\vec{v}$$

$$= \frac{1}{2}m\int_A^B \frac{d}{dt}(\underbrace{\vec{v}\cdot\vec{v}}_{v^2})\,dt$$

$$= \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

Define $K \equiv \frac{1}{2}mv^2$ → "kinetic energy"

So:

$$\underbrace{\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2}_{(W_{A\to B})} = \underbrace{mgh_A - mgh_B}_{(W_{A\to B})}$$

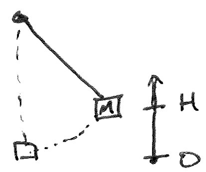
$$\Rightarrow \quad \underbrace{\frac{1}{2}mv_B^2 + mgh_B}_{E_B} = \underbrace{\frac{1}{2}mv_A^2 + mgh_A}_{E_A}$$

"energy": a thing we can calculate, and it is a constant as the system evolves.

Two parts: kinetic (depends on velocity) plus potential (depends ~~on~~ position)

$$K_A + U_A = K_B + U_B$$

Pendulum:



$K_A = 0$

$U_A = mgH$

$K_B = ? = \frac{1}{2} m v_B^2$

$U_B = mg(0) = 0$

$$\Rightarrow \quad mgH = \frac{1}{2} m v_B^2$$

$$\Rightarrow \quad v_B = \sqrt{2gH}$$

[Demo with pendulum & photogate]

→ demo with peg & pendulum

→ demo with "wrecking ball"

Other forces?

spring: $\vec{F} = -k\vec{x} \Rightarrow U(x) = -\frac{1}{2}kx^2 + \text{const}$

gravity: $\vec{F} = -G\frac{mM}{r^2}\hat{r} \Rightarrow U(r) = -G\frac{mM}{r}$

Escape velocity: did not get to in class

If $w_{A\to B}$ is path dependent, cannot define U.

→ "non-conservative" force

e.g. friction, drag.

E is still conserved globally; we just aren't keeping track of all the individual molecules, etc.

K.E. $< mgh \Rightarrow$ energy lost due to work done by friction:

$$w_f = -(\mu F_N d)$$

Units

$$\left.\begin{array}{l}(F)(d)\\ \frac{1}{2}mv^2\\ mgh\\ \vdots\end{array}\right\}\ N\cdot m \equiv \text{joule } (J)$$

1 calorie = 4.184 J

1 food calorie = 1 kcal = 4.184 kJ

In food:

carbs: ~4 kcal/g

fats: ~9 kcal/g

gasoline: ~11 kcal/g

Power

$\frac{dW}{dt}$ = power , $\frac{J}{s} \equiv$ watt (W) $\qquad$ 1 hp = 746 W

demo: up the lecture hall $\Rightarrow$ ~1 hp

on average:

human E_{in} = 2000 kcal/day = E_{out}

$\Rightarrow$ 100 W of power (mostly heat) from a human.

Aside on jumping:

Animal jumping: release energy from muscles to create motion

$$E_{released} \propto \text{muscle mass} \propto \text{mass}$$

$$\Rightarrow \quad E_{rel} = \beta M$$

height?

$$\Delta U = Mgh_{max} = E_{rel}$$

$$\Rightarrow \quad Mgh_{max} = \beta M$$

$$\Rightarrow \quad h_{max} = \frac{\beta}{g} \rightarrow \text{no } M!$$

$\Rightarrow$ all animals jump the same height, regardless of size (neglecting "engineering" differences)