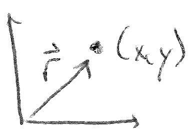
Ph1a Lecture 6 10/14/11

- Uniform circular motion
- Central forces
- Gravity
  - ↳ Circular orbits

Uniform circular motion



represent object's location vectorially:

$$\vec{r} = x\hat{x} + y\hat{y}$$

$(\hat{i})$ $(\hat{j})$ ← alternative notation

Circular motion: $r = \text{const.}$

$\theta \neq \text{const.}$

"Uniform" : $\frac{d\theta}{dt} = \text{const} \equiv$ angular velocity (or speed) $\equiv \omega$ "omega"

$\Rightarrow \quad \theta(t) = \omega t + \theta_0$

{in analogy with $x(t) = vt + x_0$} ← linear motion

One complete loop: $\theta \to \theta + 2\pi$ in time T

$\Rightarrow \quad \omega = 2\pi/T$ ← "period" T

Take $\theta_0 = 0$:

$$\vec{r}(t) = (r\cos\omega t)\hat{x} + (r\sin\omega t)\hat{y}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = (-r\omega\sin\omega t)\hat{x} + (r\omega\cos\omega t)\hat{y}$$

$\vec{v} \perp \vec{r}$ from: picture

: $r = \text{const}$

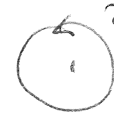
: $\vec{v} \cdot \vec{r} = vr\cos\theta = 0$

$$\vec{v} \cdot \vec{r} = -r^2\omega \cos\omega t \sin\omega t + r^2\omega \cos\omega t \sin\omega t$$

$$= 0 \checkmark$$

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2} = \omega r$$

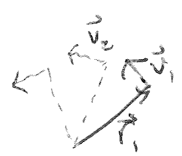
$$\boxed{v = \omega r}$$

Also:  $D = 2\pi r$ in time T

$\Rightarrow v = \frac{2\pi r}{T}$

$\Rightarrow v = \omega r$

Accel?



$\vec{v}_1 \neq \vec{v}_2 \Rightarrow \vec{a} \neq 0$

$$\vec{a} = (-r\omega^2 \cos\omega t)\hat{x} + (-r\omega^2 \sin\omega t)\hat{y}$$

$$\vec{a}(t) = -\omega^2 \vec{r}(t) \qquad \vec{a} \parallel \vec{r}$$

and

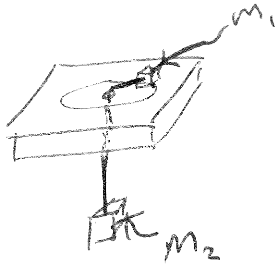
$$\boxed{a = \omega^2 r}$$

$\vec{a}$ is needed; $\vec{F}_{centrip.}$ provides: $\vec{F}_c = m\vec{a} = -m\omega^2\vec{r}$

or

$$F_c = m\omega^2 r = mv^2/r$$

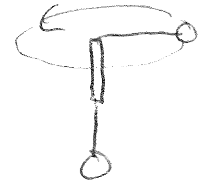
any (net) force can be the centripetal force.



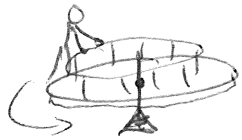
$$m_2 g = m_1 \omega^2 r$$

$$r = \frac{m_2}{m_1 g}\left(\frac{T}{2\pi}\right)^2$$

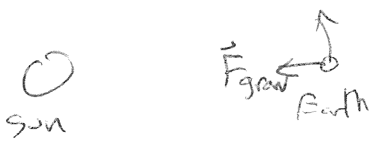
$\Rightarrow$ Demo



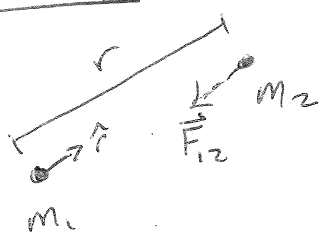
$r \propto T^2$



Many cases of circ. motion with different sources of $\vec{F}_{centripetal}$



Gravity



Newton's insight:

$$\vec{F}_{12} \propto \frac{m_1 m_2}{r^2}$$

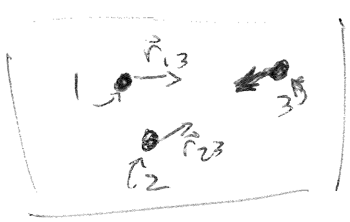
or

$$\boxed{\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}}$$

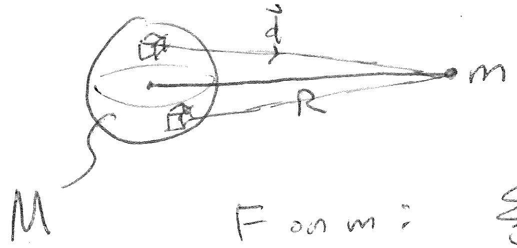
$\hookrightarrow G = 6.67 \times 10^{-11}\ N m^2/kg^2$

This plus $\vec{F} = m\vec{a} \Rightarrow$ powerful!

More objects?



$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$



F on m: $\sum_i \vec{F}_i \rightarrow \int (-Gm) \frac{\rho(\vec{r})\,dV}{|\vec{d}(\vec{r})|^2} \hat{d}(r)$ ← UGLY!

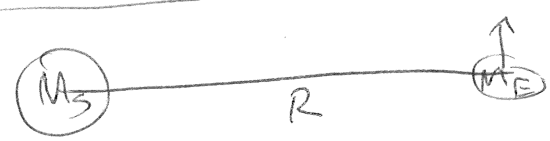
(density ρ at $\vec{r}$)

If $\rho(\vec{r}) = \rho(r)$ {spherically symmetric object}
then

$$\vec{F}_{net} = -G \frac{Mm}{R^2} \hat{R}$$ ← as if all mass at center.

Handy!

Earth around sun:



$\omega = \frac{2\pi}{1\,yr}$

$$\underbrace{m_E \omega^2 R}_{F_{centrip.}} = \underbrace{\frac{G M_S m_E}{R^2}}_{F_{grav.}}$$

$$\Rightarrow R^3 = \frac{G M_S}{\omega^2} = G M_S \left(\frac{T}{2\pi}\right)^2$$

↑ no m_E!

Kepler's 3rd Law
$R^2 \propto T^3$

Aside: orbit = falling

m LEO (low earth orbit) ~300km up

"g" ~ 8.8 m/s²

☆ | more on LEO & gravity: PDF slides →

So, for satellites orbiting earth…

$$R = \left[G M_{\oplus} \left(\frac{T}{2\pi} \right)^2 \right]^{\frac{1}{3}}$$

	Altitude	*R*	*T*

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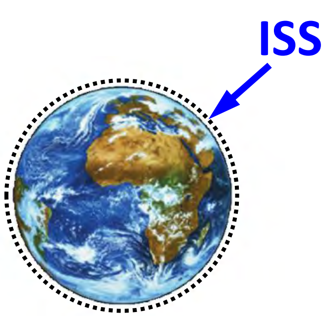

DIAGRAM TO SCALE

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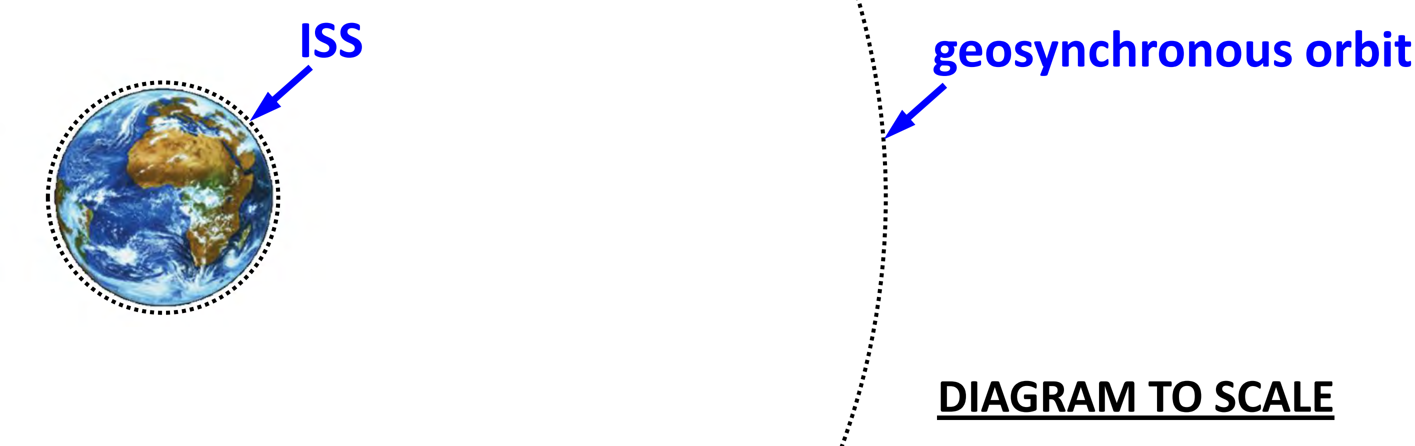

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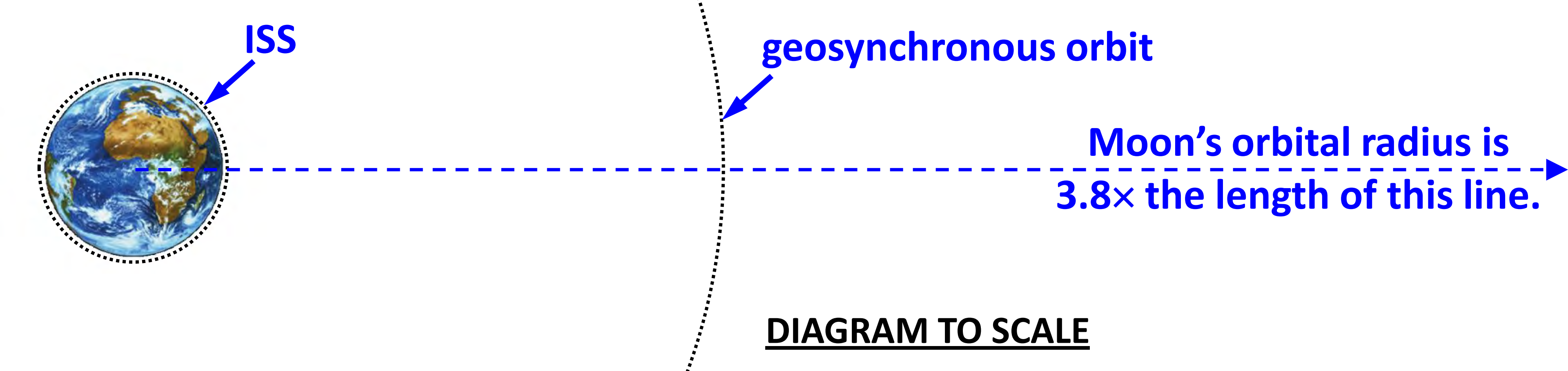

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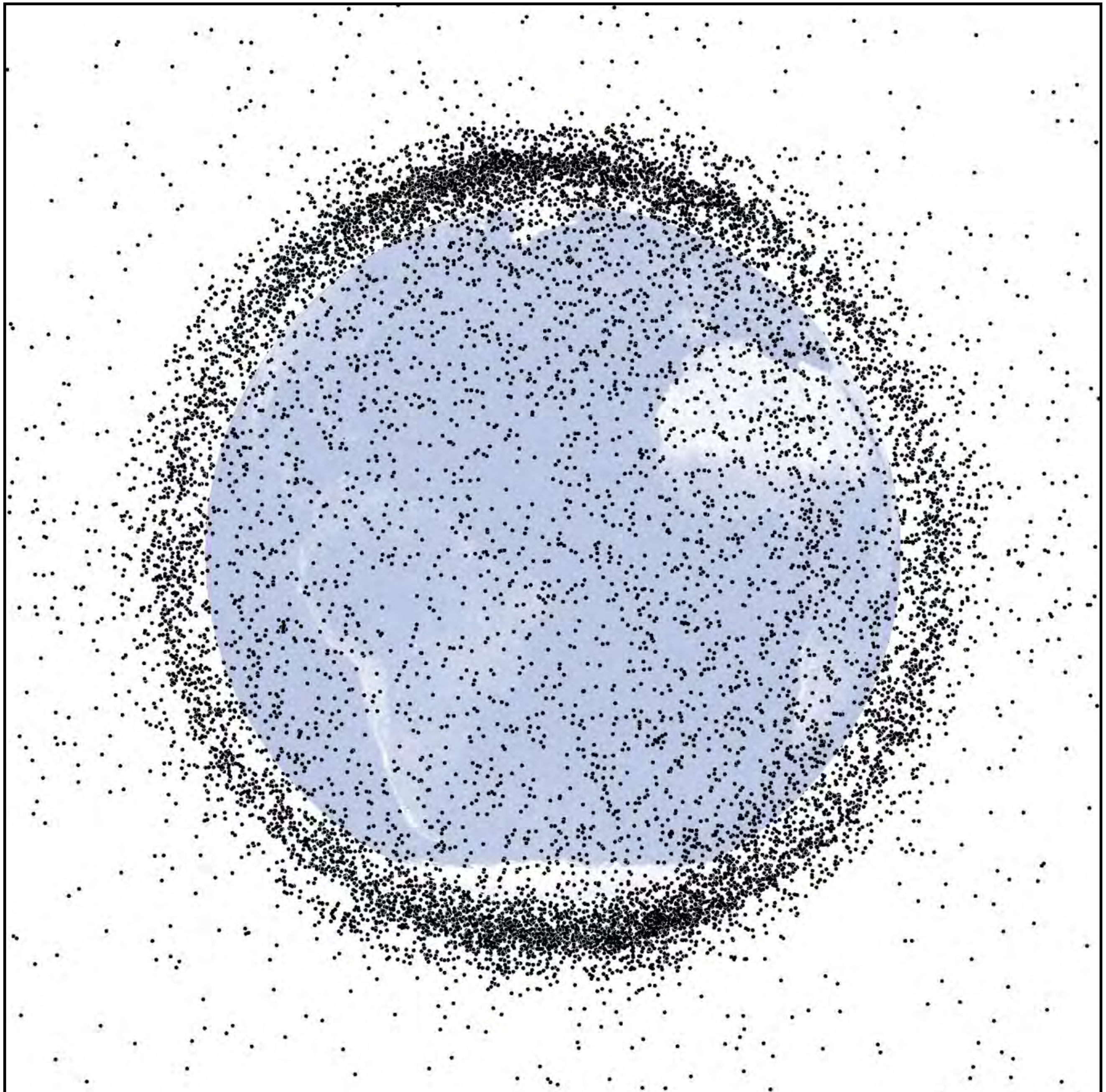
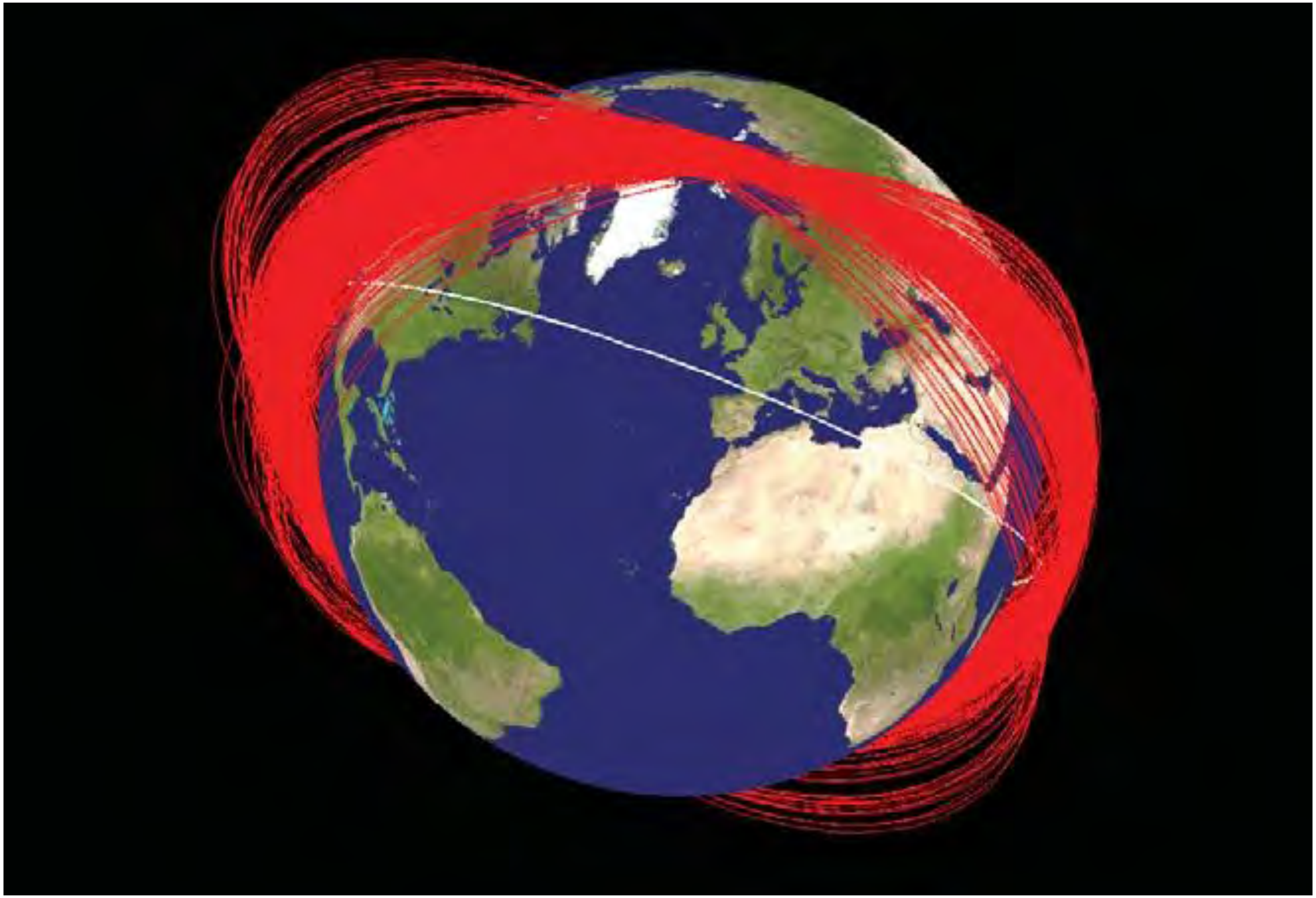
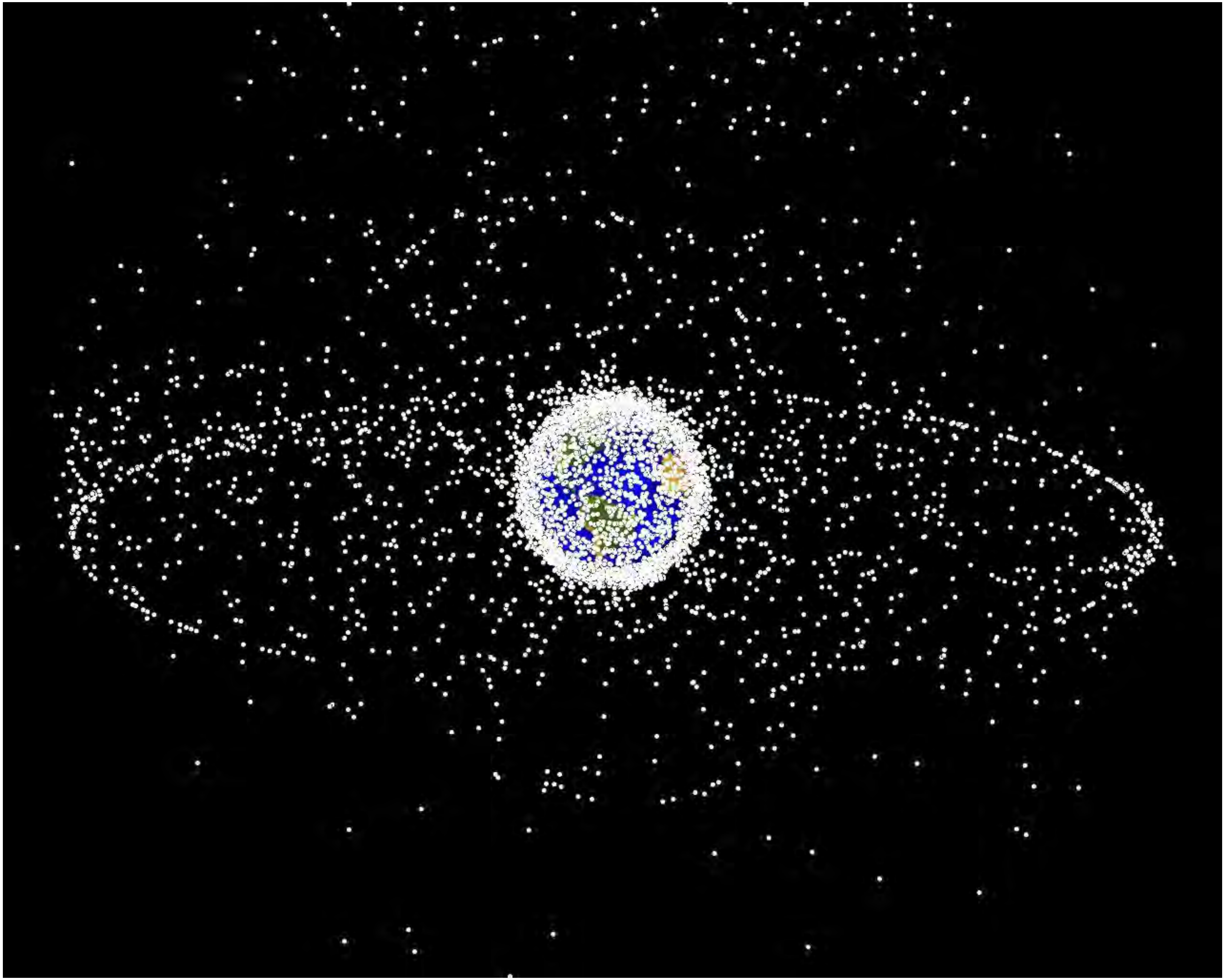
Tracked manmade objects in low earth orbit (mostly debris)

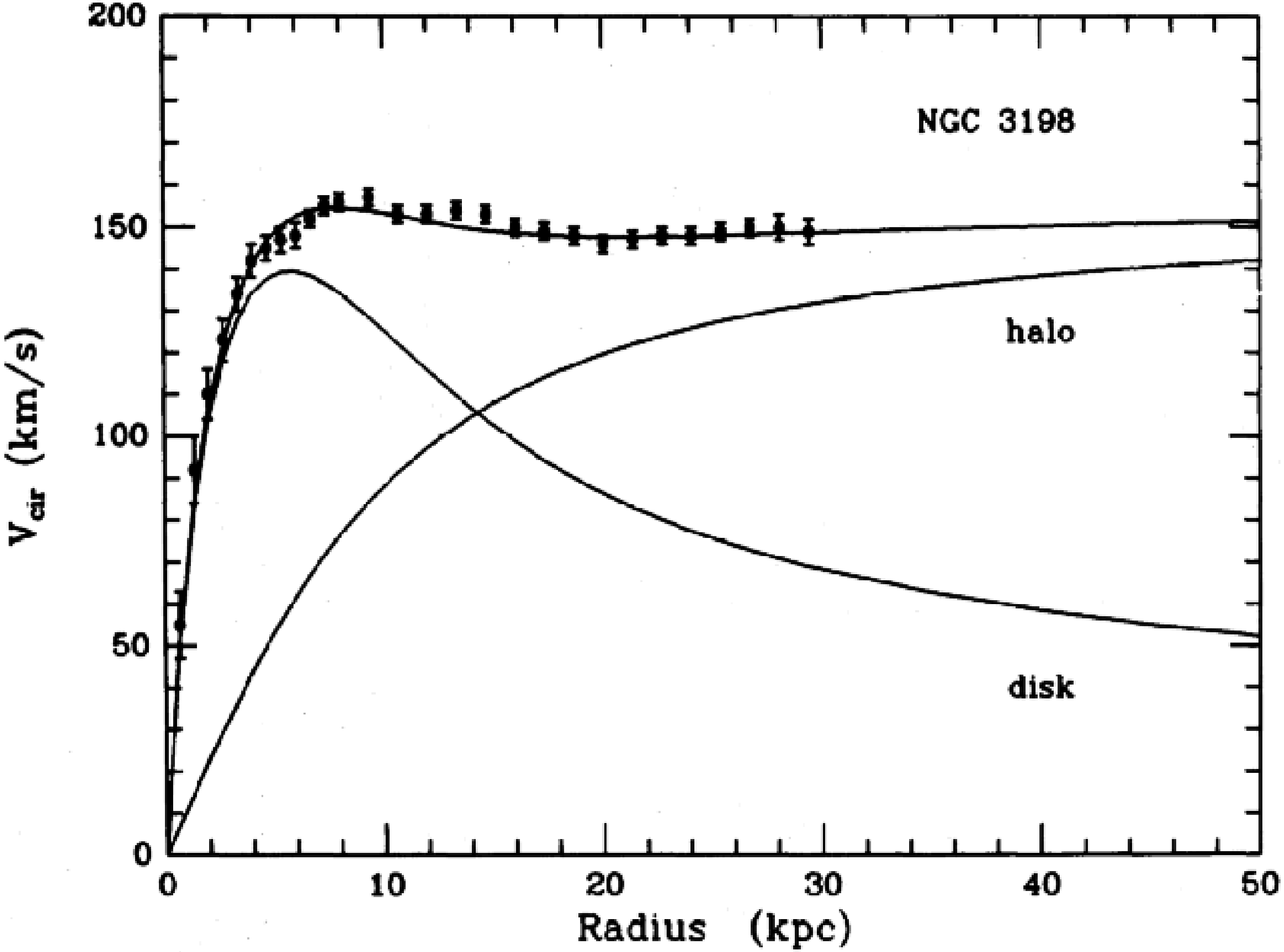


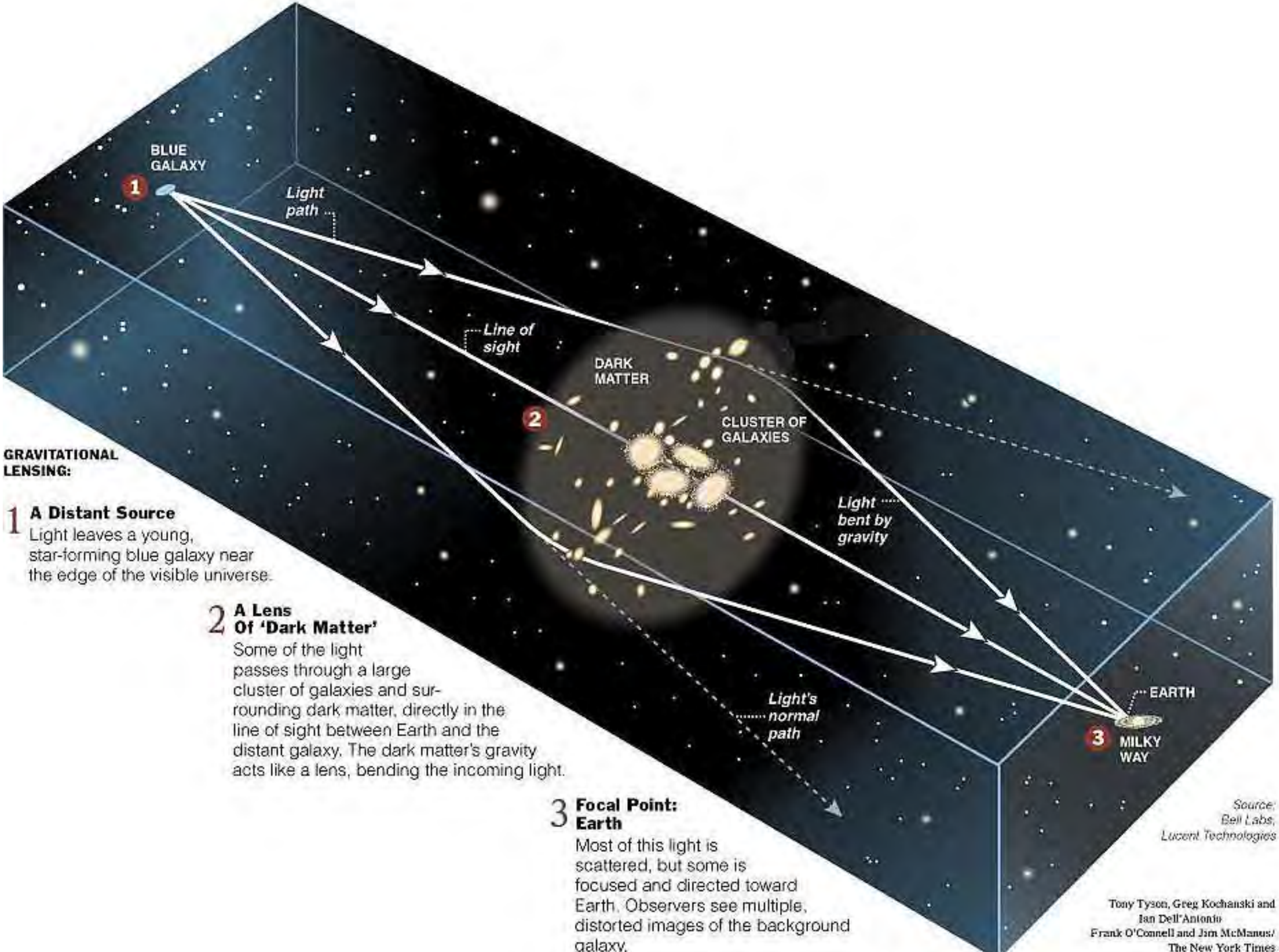


2,317 trackable objects produced in a Chinese anti-satellite missile test in 2007.



DISTRIBUTION OF DARK MATTER IN NGC 3198





GRAVITATIONAL LENSING:

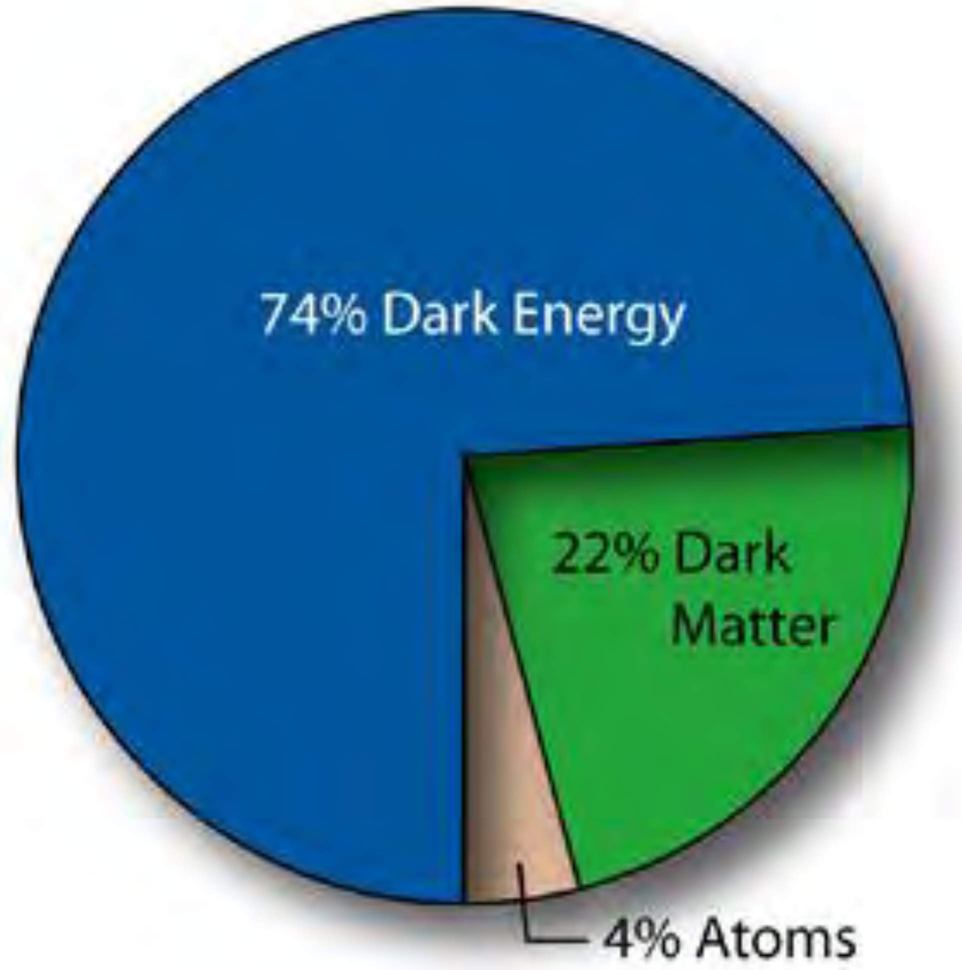
1 **A Distant Source**
Light leaves a young, star-forming blue galaxy near the edge of the visible universe.

2 **A Lens Of 'Dark Matter'**
Some of the light passes through a large cluster of galaxies and surrounding dark matter, directly in the line of sight between Earth and the distant galaxy. The dark matter's gravity acts like a lens, bending the incoming light.

3 **Focal Point: Earth**
Most of this light is scattered, but some is focused and directed toward Earth. Observers see multiple, distorted images of the background galaxy.

Source: Bell Labs, Lucent Technologies

Tony Tyson, Greg Kochanski and Ian Dell'Antonio
Frank O'Connell and Jim McManus/ The New York Times


74% Dark Energy
22% Dark Matter
4% Atoms