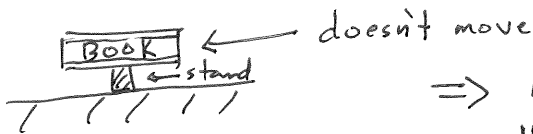


Reference frames, inertia  
2D motion, projectiles

Reference frames

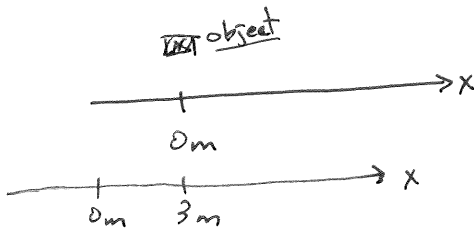


=> Let's invent a rule about nature:

→ || Something at rest stays at rest unless you do something to it.

Another sensible rule:

→ || Laws of physics don't depend on choice of coordinate system.



← case 1:  $x(t) = 0\text{m}$

← case 2:  $x(t) = 3\text{m}$

} both valid descriptions

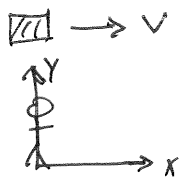
Finally, a less obvious one (perhaps):

|| Laws of physics don't depend on relative velocities of system and observer.  
↑ "coordinate system"

Together, these imply:

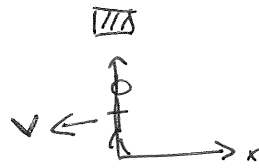
|| An object moving with constant velocity will continue to do so unless acted upon.  
→ Inertia

This is because any object moving at constant velocity can be tracked by an observer moving with it. To that observer, the object is at rest and must stay at rest.



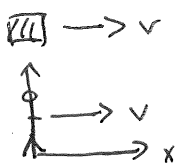
$x(t) = vt$

equivalent to  $\longleftrightarrow$



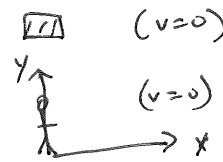
$x(t) = v(t)$

[relative velocity of v.]



$x(t) = 0$  (const)

$\longleftrightarrow$

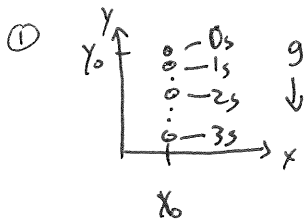


$x(t) = 0$

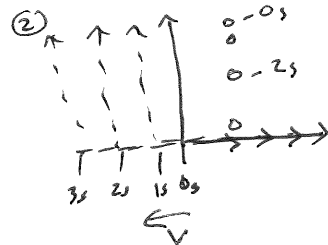
[no relative velocity]

2D motion

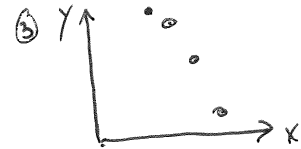
Consider relative motion that is perpendicular to objects motion:



"static" observer

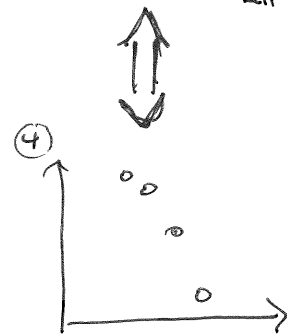


"moving" observer (coord. syst. moves with observer)



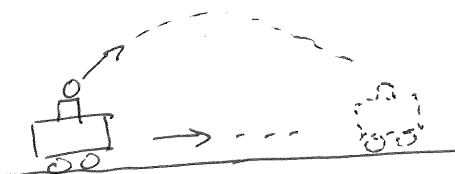
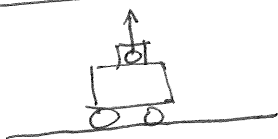
"moving" observer, but keeping coordinate system held in place  $\Rightarrow x_{ball}(t) \neq \text{const.}$

"static" and "moving" are relative concepts. Can only say that the ball and observer are moving relative to one another. (in the last three panels here...)



equivalent to: static observer watching ball with  $v_x \neq 0$

Demo: cart/ball



video capture to show parabolic path.

Trajectory

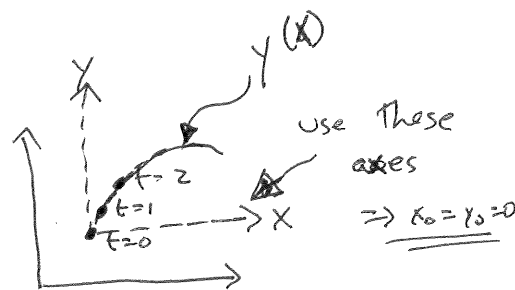
Some system in 1D:

$$y(t) = \frac{1}{2}gt^2 + v_{y0}t + y_0$$
 ← From before

Place it into motion at  $v_{x0}$ :

$x(t) = v_{x0}t + x_0$

Can set coord. system as we wish, so:



Thus,

$$\begin{cases} y(t) = \frac{1}{2}gt^2 + v_{y0}t \\ x(t) = v_{x0}t \end{cases}$$

What is  $y(x)$ ?

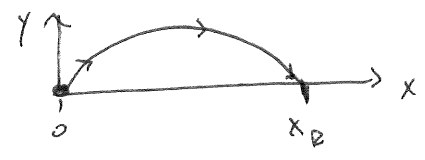
$x = v_{x0}t \Rightarrow t = \frac{x}{v_{x0}}$

$$\Rightarrow y = \frac{1}{2}g \left( \frac{x}{v_{x0}} \right)^2 + v_{y0} \left( \frac{x}{v_{x0}} \right)$$

$$y(x) = \left[ \frac{1}{2} \frac{g}{v_{x0}^2} \right] x^2 + \left[ \frac{v_{y0}}{v_{x0}} \right] x$$
 ← a parabola

Can ask any question about path.

\* When does  $y=0$ ?



$y(0) = 0$   
 $y(x_R) = 0$

Can find these "zeros":

$$y=0 = \left[ \frac{1}{2} \frac{g}{v_{x0}^2} x + \frac{v_{y0}}{v_{x0}} \right] x$$

$x=0$

$$x_R = \frac{-2v_{x0}v_{y0}}{g}$$

$\Rightarrow y=0$

$$x_R = \frac{-2v_{x0}v_{y0}}{g}$$

\* What launch angle gives max.  $x_R$ ?



$v_{x0} = v_0 \cos \theta$   
 $v_{y0} = v_0 \sin \theta$

$$\Rightarrow x_R = \frac{-2v_0^2 \cos \theta \sin \theta}{g}$$

$$= \frac{-v_0^2 \sin 2\theta}{g}$$

$\uparrow$  negative

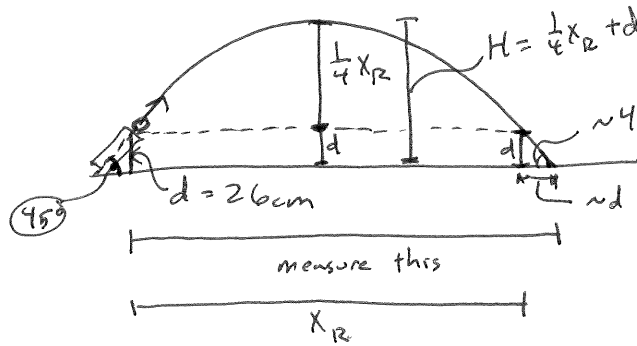
$$\Rightarrow \sin 2\theta = 1 \Rightarrow \theta = 45^\circ$$

\* How high at peak?

$$\frac{dy}{dx} = 0 \Rightarrow x_{\text{peak}} = \frac{-v_{x0}v_{y0}}{g}$$

$$y(x_{\text{peak}}) = \frac{-v_{y0}^2}{2g} = \frac{1}{4} x_R \quad (\text{if } \theta = 45^\circ)$$

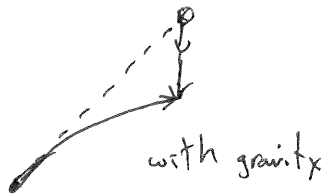
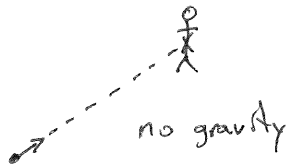
# Ring shot demo



(good approx. since  $d \ll X_R$ )

- ① Measure distance ball goes
- ② Correct for  $d$  & determine  $H$
- ③ Set up ring at  $(x = \frac{1}{2} X_R, y = H)$
- ④ Fire!

# Monkey shoot demo



gravity:  
 adds  $\frac{1}{2}gt^2$  to both  
 objects'  $y(t)$  functions  
 $\Rightarrow$  will still  
 collide.