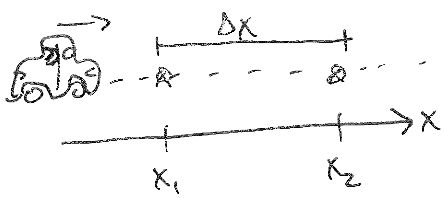
Ph 1a Lecture 2 9/30/11

Position, velocity, derivatives

Free fall / constant accel.

(& integration lite)

Position & velocity

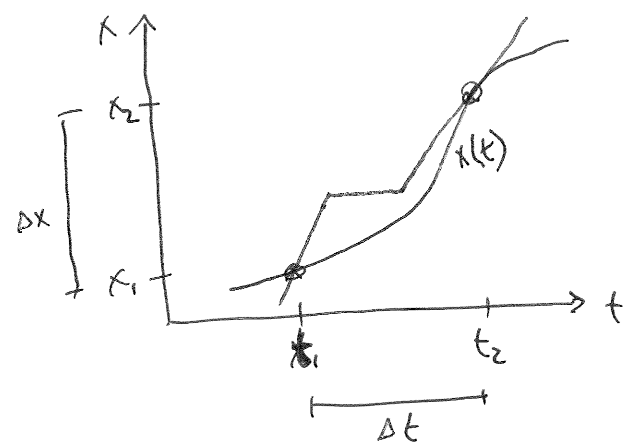


Sidebar on Δ-notation and overloaded "x" notation...

car passes x_1 at time t_1

" " x_2 " " t_2

$\hookrightarrow x_1 + \Delta x$ $\hookrightarrow t_1 + \Delta t$



Velocity ≡ rate of change in position

Avg. vel. over $[t_1, t_2]$ interval

"avg" → $\bar{v} = \dfrac{\Delta x}{\Delta t}$

what's v <u>at</u> t_1 : $v(t_1)$?

Can move t_2 closer and closer to t_1 $\Leftrightarrow$ $\Delta t \to 0$

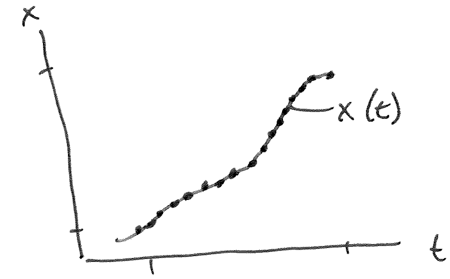
$$v(t_1) = \underbrace{\lim_{\Delta t \to 0} \frac{\overbrace{x(t_1 + \Delta t) - x(t_1)}^{\Delta x}}{\Delta t}}_{\text{derivative of } x(t) \text{ w.r.t. } t}$$

$$= \frac{d}{dt} x(t) = \frac{dx}{dt} = \dot{x} = \dot{x}(t) = \ldots$$

If we could measure car's location twice in quick succession, could get good estimate of $v(t_1)$.

(GPS / satnav system does this)

→ Leap to concept that continuous $x(t)$ exists →



Assume $x(t) = ct$ ← constant

what is $v(t)$ = veloc. at any time t?

$$v(t) = \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{\cancel{ct} + c\cancel{\Delta t} - \cancel{ct}}{\cancel{\Delta t}}$$

$$= c$$

Can tabulate this limit-taking process:

$$\frac{d}{dt}(c) = 0$$

$$\frac{d}{dt}(ct) = c$$

$$\frac{d}{dt}(ct^2) = 2ct$$

$$\frac{d}{dt}(ct^n) = nct^{n-1}$$

$$\vdots$$

$$\frac{d}{dt}(\sin ct) = c \cos ct$$

$$\vdots$$

Sum rule:

$$\frac{d}{dt}(x+y) = \frac{dx}{dt} + \frac{dy}{dt}$$

Prod. rule:

$$\frac{d}{dt}(xy) = x\frac{dy}{dt} + \frac{dx}{dt}y$$

Chain rule:

$$\frac{d}{dt}(f(g(t))) = \frac{df}{dg}\frac{dg}{dt}$$

Position: x

Velocity: rate of change in position : $\frac{dx}{dt} = v(t)$

Accel. : " " " " velocity : $\frac{dv}{dt} = a(t)$

Freefall

Galileo's claim that $a = \text{const.}$ is non-intuitive

→ drop some things

→ vacuum demo

$$a(t) = a \equiv g \quad \text{on Earth}$$

$$= -9.8 \ m/s^2$$

neg. if position axis points up.

If $a(t) = a_0 = \frac{dv}{dt}$, what's $v(t)$?

Look at deriv. table...

$$\frac{d}{dt}(a_0 t) = a_0 \quad ✓$$

But:

$$\frac{d}{dt}(a_0 t + v_0) = a_0 \quad ✓$$

$\boxed{v(t) = a_0 t + v_0}$ works

$v(t=0) = 0 + v_0$

$= v_0$ ← initial velocity

"Antiderivative"

$$\int a_0 \, dt = a_0 t + (\text{constant})$$

Do it again for $v(t) \to x(t)$.

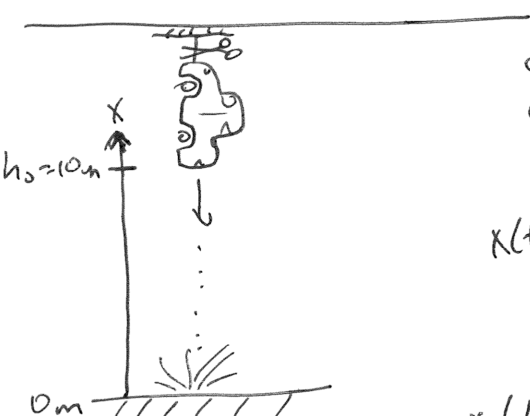
$$\boxed{x(t) = \frac{1}{2} a_0 t^2 + v_0 t + x_0}$$

Test:

$$v(t) = \frac{dx}{dt} = a_0 t + v_0$$

Also:

$$x(t=0) = 0 + 0 + x_0$$
$$= x_0 \checkmark$$



cut rope at $t=0$.
when does the car hit the floor?

$$x(t) = \underbrace{\frac{1}{2} g}_{-9.8\,m/s^2} t^2 + \underbrace{0}_{v_0=0} \cdot t + h_0$$

$$x(t_c) = 0 \Rightarrow t_c = ?$$

Solve: $0 = \frac{1}{2} g t_c^2 + h_0$

$$\Rightarrow t_c^2 = \sqrt{\frac{-2h_0}{g}}$$

For us: $\sqrt{\frac{-2 \times 10m}{-9.8\,m/s^2}}$

$= 1.43\,s$

How fast at impact? $\Leftrightarrow v(t_c) = ?$

$$v(t_c) = g t_c$$
$$= (-9.8\,m/s^2)(1.43\,s) = 14.0\,m/s = 31.3 \text{ mph}$$

wear your seatbelt, even at "low" speed!

Ball drop demo

$$x(t_c) = h_0 + \frac{1}{2} g t_c^2$$

will measure t_c!

$$\underline{\underline{g = \frac{-2h_0}{t_c^2}}}$$

Can calculate g using h_0, t_c.

Table of results

h_0	t_c	$\Rightarrow$	g
50cm	.		.
100cm	.		.
⋮	⋮		⋮