

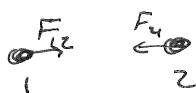
Momentum conservation

- ↳ 2 objects
- ↳ N "

Center of mass

Some examples

Momentum



$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\Rightarrow \frac{d}{dt}(m_1 \vec{v}_1) = -\frac{d}{dt}(m_2 \vec{v}_2)$$

$$\Rightarrow \frac{d}{dt}(\underbrace{m_1 \vec{v}_1 + m_2 \vec{v}_2}_{\vec{P}}) = 0$$

"momentum"

$$\boxed{\vec{P}_1 + \vec{P}_2 = \text{const}}$$

Example



initial:

final:

$$\left. \begin{aligned} P: & \quad mv + 0 = mv_1 + mv_2 \\ KE: & \quad \frac{1}{2}mv^2 + 0 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \end{aligned} \right\}$$

Two solns:

$$\boxed{\begin{aligned} v_1 = 0, v_2 = v \\ \text{OR} \\ v_1 = v, v_2 = 0 \end{aligned}}$$

Elastic collision: $\Delta KE = 0$

Demo: track collisions
Newton's cradle

N particles



$$\vec{F}_i = \vec{F}_i^{\text{ext}} + \sum_{\substack{j=1 \\ j \neq i}}^N \vec{F}_{ij} = \frac{d}{dt} \vec{p}_i \quad m_i \vec{v}_i$$

Sum over all i :

$$\vec{F}_{\text{tot}} = \sum_{i=1}^N \vec{F}_i = \sum_{i=1}^N \vec{F}_i^{\text{ext}} + \underbrace{\sum_{i=1}^N \left[\sum_{\substack{j=1 \\ j \neq i}}^N \vec{F}_{ij} \right]}_{\substack{\vec{F}_{ij} = -\vec{F}_{ji} \\ \Rightarrow 0!}} = \frac{d}{dt} \sum \vec{p}_i$$

$$\vec{F}_{\text{tot}} = \vec{F}^{\text{ext}} = \frac{d}{dt} \vec{P}_{\text{tot}}$$

$$\text{if } \vec{F}^{\text{ext}} = 0, \quad \vec{P}_{\text{tot}} = \text{const}$$

↳ conservation of momentum

$$\vec{F}_{\text{tot}} = \frac{d}{dt} [M \vec{v}]$$

↳ \vec{v} of what?

center of mass...

Center of mass

First: demo using $[2m] [m]$ ← "blasting" cap

$$\vec{r}_c = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{1}{M} \sum m_i \vec{r}_i = \text{center of mass}$$



$$\vec{v}_c = \frac{1}{M} \sum m_i \vec{v}_i = \frac{1}{M} \vec{P}_{\text{tot}} \Rightarrow \underline{\underline{M \vec{v}_c = \vec{P}_{\text{tot}}}} \quad \checkmark$$

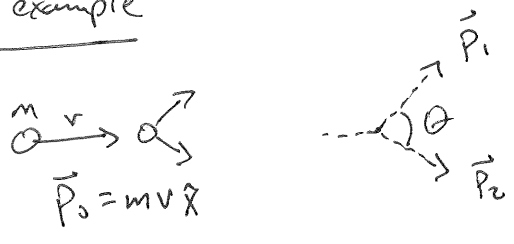
Also: $\vec{F}_{\text{tot}} = M \vec{a}_c$

→ CoM moves as if total force on object acts through CoM
 ("solid" object or not (eg. solar system))

If $\vec{F}_{tot} = 0, \vec{a}_c = 0 \Rightarrow \vec{v}_c = \text{const.}$

⇒ "CoM frame", in which $\vec{P}_{tot} = 0$

2D example



$\theta = ?$

P: $\vec{P}_0 = \vec{P}_1 + \vec{P}_2$

⇒ $\begin{cases} P_0^2 = P_1^2 + P_2^2 + 2\vec{P}_1 \cdot \vec{P}_2 \\ P_0^2 = P_1^2 + P_2^2 \end{cases}$

K: $\frac{P_0^2}{2m} = \frac{P_1^2}{2m} + \frac{P_2^2}{2m}$



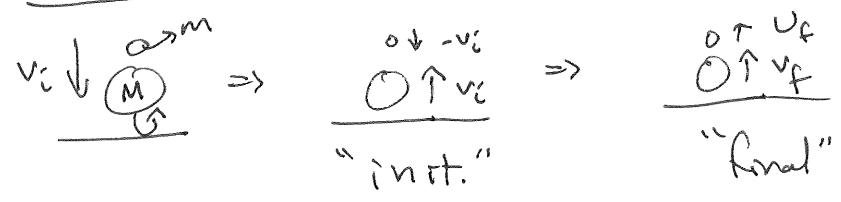
$2\vec{P}_1 \cdot \vec{P}_2 = 0 \Rightarrow \cos\theta = 0$
 $\Rightarrow \theta = 90^\circ$

Aside:
 $K = \frac{1}{2}mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{P^2}{2m}$

[Demo: 2D collision with pool balls]

θ slightly less than $90^\circ \Rightarrow$ K.E. lost

"Cannon"



(4)

$$\left. \begin{array}{l} P: (M-m)v_i = mU_f + Mv_f \\ K: \frac{1}{2}Mv_i^2 + \frac{1}{2}mv_i^2 = \frac{1}{2}mU_f^2 + \frac{1}{2}Mv_f^2 \end{array} \right\} U_f = ?$$

(1) from (P): $v_f = \frac{1}{M} [(M-m)v_i - mU_f]$

(2) plug v_f into (K)

(3) algebra

$$(4) \Rightarrow U_f^2 + 2 \left[\frac{m}{M} - 1 \right] v_i U_f + \left[\frac{m}{M} - 3 \right] v_i^2 = 0$$

For simplicity, take $\frac{m}{M} \rightarrow 0$ (small m , large M)

$$U_f^2 - 2v_i U_f - 3v_i^2 = 0$$

$$\Rightarrow U_f = \frac{2v_i \pm \sqrt{4v_i^2 + 4(3v_i^2)}}{2}$$

$$\Rightarrow U_f = v_i [1 \pm 2] = \begin{cases} -v_i & \text{OR} & \leftarrow \text{mit. case} \\ 3v_i & & \leftarrow \text{post-collision} \end{cases}$$

\Rightarrow KE is 9x more than simple bounce

\Rightarrow h_{\max} is 9x more (mgh)

[Demo: "cannon"]