Ph1a Homework Solution 4

Fall 2015

Each homework problem is worth 5 points. Please disregard the point values listed on the problem itself. Use these instead.

4.1 Frautschi 10.21 (5 points)

4.1.a (5 points)

Suppose the child leaves the mound at some angle $\theta$ (as indicated in the question). We know by conservation of energy (assume no frictional loss), that at $\theta$,

\[ E = T + V = \frac{1}{2}mv^2 + mgh' = E_0 = mgR \]

\[ \implies v^2 = 2(gR - gh') = 2gR(1 - \sin(\theta)) \]

where $h' = R \sin(\theta)$ is the height above ground when the child leaves the mound. Additionally, because the child is just about to lose contact with the mound at this point, the normal force exerted by the mound on the child must also be 0. That is, the component of the gravitational acceleration that points toward the center of the semi-circle must also be equal to the centripetal acceleration of the child $a_c$. Therefore,

\[ a_c = \frac{v^2}{R} = g \sin(\theta) \]

\[ \implies v^2 = gR \sin(\theta) \]

Combine the 2 equations above, we can solve for theta.

\[ gR \sin(\theta) = 2gR(1 - \sin(\theta)) \]

\[ \sin(\theta) = \frac{2}{3} \]

\[ \theta = \arcsin(2/3) \approx 0.73[rad] \approx 41.8[deg] \]

4.2 QP7 (5 points)

4.2.a (1 point)

The total kinetic energy just before the collision is equal to the total potential energy before the masses are released.

\[ K = U_m + U_M = (m + M)gh \]

4.2.b (2 points)

For inelastic collisions, momentum is conserved and kinetic energy is not conserved. Define $v_0 = \sqrt{2gh}$ and the velocity after the collision as $v_f$. Conservation of momentum gives

\[ mv_0 + M(-v_0) = (m + M)v_f \]

\[ v_f = \frac{m - M}{m + M}v_0 \]

The resulting kinetic energy ($K_f$) is

\[ K_f = \frac{1}{2}(m + M)v_f^2 = \frac{(m - M)^2}{(m + M)}gh. \]
Defining the maximum height after the collision as \( h' \), conservation of energy (after the collision) gives

\[
(m + M)gh' = \left(\frac{m - M}{m + M}\right)^2 (m - M)gh \\
h' = \left(\frac{m - M}{m + M}\right)^2 h
\]

4.2.c (2 points)

Elastic collisions conserve both momentum and kinetic energy. Defining the velocities of the masses after the collision as \( v_m \) and \( v_M \) gives final momentum and kinetic energy of

\[
p_f = mv_m + Mv_M \\
K_f = \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2
\]

The conservation laws give

\[
(m - M)v_0 = mv_m + Mv_M \\
\frac{1}{2}(m + M)v_0^2 = \frac{1}{2}mv_m^2 + \frac{1}{2}Mv_M^2
\]

Solving these equations for the velocity of the small mass

\[
v_m = \frac{m - 3M}{m + M}v_0 \\
K_m = \frac{1}{2}mv_m^2 = \left(\frac{m - 3M}{m + M}\right)^2 (mgh)
\]

Conserving energy for the small mass after the collision gives the maximum height \( h' \)

\[
mgh' = \left(\frac{m - 3M}{m + M}\right)^2 (mgh) \\
h' = \left(\frac{m - 3M}{m + M}\right)^2 h
\]

4.3 QP50 (10 points)

4.3.a (1 point)

By conservation of energy, we have

\[
\frac{1}{2}mv^2 = mgh \\
v = \sqrt{2gh}
\]

since speed \( v \) is always defined to be positive.

4.3.b (2 points)

The block experiences both friction and a restoring force from the spring, namely

\[
F = F_f + F_s \\
= -\mu mg - kx
\]
4.3.c  (2 points)
Total work done on the block is given by

\[ W = \int_0^{x_s} F(x) dx = \int_0^{x_s} (-\mu mg - kx) dx = -\mu mgx_s - \frac{1}{2}kx_s^2 \]  

(12)

It is negative because the block was losing energy in this stage.

4.3.d  (2 points)
The total energy dissipated by friction upon returning to \( x = 0 \) is simply

\[ W_f = 2|F f x_s| = 2\mu mgx_s \]  

(13)

4.3.e  (2 points)
The total energy upon returning to the point \( x = 0 \) is then given by

\[ E = E_0 - W_f = mgh - W_f \]  

(14)

Thus the maximum height \( h' \), by energy conservation, is

\[ E = mgh - 2\mu mgx_s = mgh' \]  

(15)

\[ \implies h' = h - \frac{W_f}{mg} = h - 2\mu x_s \]  

(16)

4.3.f  (1 point)

\[ x_s = \frac{h - h'}{2\mu} \]  

(17)

4.4  FP2 (5 points)

4.4.a  (2 points)
Prior to the collision, the total energy and linear momentum of the two-mass system is

\[ E_{1i} = \frac{1}{2}m_1v_1^2 = \frac{1}{2}mv^2 \]  

\[ E_{2i} = \frac{1}{2}m_2v_2^2 = \frac{1}{2}(ab^2)mv^2 \]  

\[ E_i = \frac{1}{2}mv^2 \left[ 1 + ab^2 \right] \]

\[ \vec{p}_{1i} = m_1\vec{v}_1 = m(v\hat{x}) \]  

\[ \vec{p}_{2i} = m_2\vec{v}_2 = (am) \left[ b(v\cos\theta\hat{x} + \sin\theta\hat{y}) \right] \]  

\[ \vec{p}_i = mv \left[ (1 + ab\cos\theta)\hat{x} + (ab\sin\theta)\hat{y} \right] \]
4.4.b (3 points)
In an inelastic collision, momentum is conserved but kinetic energy is not.

\[ \vec{p}_i = m v [(1 + ab \cos \theta) \hat{x} + (ab \sin \theta) \hat{y}] \]
\[ \vec{p}_f = (m_1 + m_2) \vec{v}_f \]
\[ \vec{v}_f = \frac{\vec{p}_f}{m(1 + a)} = \frac{\vec{p}_i}{m(1 + a)} \]
\[ \vec{v}_f = \frac{v}{1 + a} [(1 + ab \cos \theta) \hat{x} + (ab \sin \theta) \hat{y}] \]
\[ E_f = \frac{1}{2} (m_1 + m_2) v_f^2 \]
\[ = \frac{1}{2} m(1 + a) v_f^2 \]
\[ = \frac{mv^2}{2(1 + a)} [(1 + ab \cos \theta)^2 + (ab \sin \theta)^2] \]
\[ E_f = \frac{mv^2}{2(1 + a)} [1 + a^2 b^2 + 2ab \cos \theta] \]
\[ \Delta E = \frac{1}{2} mv^2 \left[ (1 + ab^2) - \left( \frac{1 + a^2 b^2 + 2ab \cos \theta}{1 + a} \right) \right] \]
\[ = \frac{1}{2} mv^2 \left( \frac{a}{1 + a} \right) (1 + b^2 - 2b \cos \theta) \]